International Journal of Scientific Research in Mathematical and Statistical Sciences
Research Paper
Vol.6, Issue.5, pp.46-51, October (2019)
E-ISSN: 2348-4519
DOI: https://doi.org/10.26438/ijsrmss/v6i5.4651

Algebraic Structures of Soft Sub Fields in View of Fuzzy Soft Environment

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Available online at: www.isroset.org

Received: 27/Aug/2019, Accepted: 16/Oct/2019, Online: 31/Oct/2019

Abstract: In this paper, we discuss the analysis of fuzzy soft sub modules over subfields of a field. Some related properties about algebraic substructures of soft fields and soft sub modules are investigated and illustrated by many examples. Finally, we discuss the correlation coefficient between them.

Keywords: *Soft set. fuzzy set, interval valued fuzzy set, soft module , soft subfield, trivial, whole, correlation, intersection, product, union.*

I. INTRODUCTION

Molodtsov [20] elaborated a lot of potential applications of soft sets in different fields including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory. Aktas and Cagman ([4], [5]) investigated the basic concepts of soft set theory and compared soft sets to fuzzy and rough sets, providing examples to clarify their differences. The complexities of modeling uncertain data in economics, engineering, environmental science, sociology, medical science and many other fields cannot be successfully dealt with by classical methods. While probability theory, fuzzy set theory([28], [29]), rough set theory([23], [24]), vague set theory [16] and the interval mathematics [6] are useful approaches to describing uncertainty, each of these theories has its inherent difficulties. Consequently, Molodtsov [20] proposed a completely new model for modeling vagueness and uncertainty, which is called soft set theory. Now, works on soft set theory are progressing rapidly. Maji et.al [21] described the applications of soft set theory and have published a detailed theoretical study on soft sets [9].They also defined and studied soft group and derived their basic properties by using Molodtsov's definition of the soft sets. Ali et.al [6] introduced some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets. Feng et.al [15] introduced and investigated soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiring homomorphisms. Acar et.al [2] introduced initial concepts of soft rings. Atagun and Sezgin [3] introduced the notions of soft near-rings, soft subnear-rings, soft (left, right) ideals, (left, right) idealistic soft near-rings and soft near-ring homomorphisms and investigated them with many corresponding examples. In [10], Cagman and Enginoglu defined soft matrices and their operations to construct a soft max–min decision making method which can be successfully applied to the problems that contain uncertainties. Sezgin et.al [27] extended the study of soft near-rings especially with respect to the idealistic soft near-rings as well. The algebraic structure of set theories dealing with uncertainties has also been studied by some authors. Rosenfeld [25] proposed the concept of fuzzy groups in order to establish the algebraic structures of fuzzy sets. Abou-Zaid [1]introduced the notion of a fuzzy subnear-ring and studied fuzzy ideals of a near-ring. This concept is also discussed by many authors (e.g., [19], [20], [21], [22]). Atagun [6] defined the notions of soft subnear-rings, soft ideals and soft N-subgroups of near-rings. Rough groups were defined by Biswas et.al [8] and some other authors (e.g., [8], [18]) have studied the algebraic properties of rough sets as well. In this paper, we discuss the analysis of fuzzy soft sub modules over subfields of a field. Some related properties about algebraic substructures of soft fields and soft sub modules are investigated and illustrated by many examples. Finally, we discuss the correlation coefficient between them.

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II. PRELIMINARIES

By a ring, we shall mean an algebraic system (R, +, •), where

(i) (R, +) forms an abelian group,
(ii) (R, •) forms a semi group and
(iii) x• (y +z) = xy + xz and (x+y) • z = xz + yz for all x,y,z ∈ R (i,e ., left and right distributive laws hold)

Through out this paper, R will always denote a ring.
A left R-module over a ring R consists of an abelian group (M , +) and an operation R × M →M such that for all r,s ∈ R, x,y ∈ M, we have

(i) r (x +y) = rx +ry
(ii) (r+s)x = rx +sx
(iii) (rs)x = r(sx). It is denoted by R^M.

Clearly R itself is a (left) R-module by natural operation. Suppose M is a left R-module and N is a subgroup of M.Then N is called a sub module (or R-sub module) if, for any n ∈ N and any r ∈ R, the product rn is in N. Molodtsov [20] defined the soft set in the following manner; Let U be an initial universe set, E be a set of parameters, P(U) be the power set of U and A is subset of E. Then A is called a soft set of U.

In fact, there exists a mutual correspondence between soft sets and binary relations as shown in [21] . That is, let A and B be non-empty sets and assume that α refers to an arbitrary binary relation between an element of A and an element of B.A set-valued function δ : A → B can be defined as δ (x) = { y ∈ B /(x, y) ∈ α }.

Definition 2.1**[Molodtsov]:** Let δ_A and δ_B be two soft sets over a common universe U such that A ∩ B ≠ ϕ. The restricted intersection of δ_A and Δ_B is denoted by $\delta_A \cap \Delta_B$, and it is denoted by $\delta_A \cap \Delta_B = H_C$, where C = A ∩ B and for all c ∈ C, H(C) = δ(C) ∩ Δ(C).
Through out this section, we denote a field by Ғ and a subfield S of Ғ by S < Ғ.

Definition 2.2**[Fuzzy set]:** Let X is a set (space). A map A: X → [0,1] is called fuzzy set in X and it is defined as A = { (x , δ_A(x)) / x ∈ X } where δ_A(x) ∈ [0,1].

Definition 2.3 **[Interval valued fuzzy set]:** Let [J] ∈ [0,1] and M = [M_L, M_U] ∈ [J], where M_L and M_U are the lower extreme and the upper extreme, respectively. For a set X, an interval valued fuzzy set [IVFS] A = { (x, M_A(x)) / x ∈X } where the function M_A : X → [J] defines the grade of membership of an element x to A, and $M_A(x)$ = [$M_{AL}(x)$, $M_{AU}(x)$] is called an interval-valued fuzzy number.

Throughout this section, we denote a module by M and a sub module (rep; ideal) N of M by N < M.

Definition 2.4: Let N be a fuzzy sub module of M and let (Ғ , N) be a soft set over M. If for all x,y ∈ N and for all r ∈ R,
(SM1) Ғ (x^m-y^m) $\dot\supseteq$Ғ(x^m) ∩Ғ(y^m) and
(SM2) Ғ(rx^m) $\dot\supseteq$Ғ(x^m), then the soft set (Ғ , N) is called a fuzzy soft sub module of M and denoted by (Ғ , N) < M or simply $Ғ_N$< M.

***Example 2.5*:** Let R = (Z_{10}, +, ∘), M= (Z_{10}, +) be a left R-module with natural operation and N_1 = {0,5} be a sub module of M. Let the soft set (Ғ, N_1) over M, where Ғ : N_1→ P(M) is a set valued function defined by Ғ(0) = (0,3,4,9} and Ғ (5) = {0,9}. Then it can be easily seen that (Ғ , N_1) < M. Let N_2 = {0,2,4,6,8} < M and the soft set (G, N_2) over M, where G : N_2→ P(M) is a set -valued function defined by G(0) = {0,2,5,7,9} and G(2) = G(4) = G(6) = G(8) = { 2,9}. Then (G , N_2) < M, too. However if we define the soft set (H, N_2) over M such that H(0) = Z_{10}, H(2) = { 1,7}, H(4) = {3,5,7}, H(6) = {1,2,8}, H(8) = {2,4,7}, then H(7 . 6) = H(2) = {1,7} ⊉ H(6) = {1,2,8}. Therefore, (H, N_2) is not a soft sub module over M.

Theorem-2.6: The intersection of two fuzzy soft sub modules of M is also a fuzzy soft sub module of M.
Proof: Since S_1 and S_2 are soft sub modules of Ғ, then $S_1 \cap S_2$ is a soft sub module of Ғ.
By definition 2.1 ,, let
$G_{S1} \cap H_{S2}$ = (G,S_1) ∩ (H,S_2) = (T , $S_1 \cap S_2$), where T(x^m) = G(x^m) ∩ H(y^m) for all x ∈ $S_1 \cap S_2$ ≠ ϕ, then for all x,y ∈ $S_1 \cap S_2$.
(FSM1) T (x^m- y^m) = G(x^m- y^m) ∩ H(x^m-y^m)
$\dot\supseteq$ (G(x^m) ∩ G(y^m)) ∩ (H(x^m) ∩ H(y^m))
= (G(x^m) ∩ H(x^m)) ∩ (G(y^m) ∩ H(y^m))
= T(x^m) ∩ T(y^m),
(FSM2) T(rx^m) = G$(rx)^m$∩ H$(rx)^m$$\dot\supseteq$(G($x^m$)∩H($x^m$))=T($x^m$) .
Therefore $G_{S1} \cap H_{S2} = T_{S1 \cap S2}$< M.

Definition-2.7: Let M_1 andM_2 be fuzzy soft sub modules and let (G , S_1) and (H, S_2) be two fuzzy soft sub modules of M_1 and M_2, respectively. The product of fuzzy soft sub modules (G, S_1) and (H, S_2) is defined as (G , S_1) x (H , S_2) = (Q ,S_1xS_2), where Q(x,y) = M(x) x G(y) for all (x,y) ∈ S_1x S_2.

Theorem-2.8:The product of two fuzzy soft sub modules of M is also a fuzzy soft sub module of M.
Proof: Since S_1 and S_2 are soft sub modules of M, then $S_1 \cap S_2$ is a soft sub module of M.
By definition 2.1 , let G_{S1}x H_{S2} = (G,S_1) x (H,S_2) = (Q , S_1xS_2),where Q(x,y) = M(x) x G(y) for all (x,y) ∈ S_1x S_2.Thern for all (x_1,y_1), (x_2, y_2) ∈ S_1 x S_2.
(FSM1)Q((x_1, y_1)^m – (x_2, y_2)^m) = Q(x_1^m-x_2^m, y_1^m-y_2^m)
= G(x_1^m-x_2^m) x H(y_1^m-y_2^m) $\dot\supseteq$(G(x_1^m) xG(x_2^m)) x (H(y_1^m) ∩ H(y_2^m))
= (G(x_1^m) xH(y_1^m)) x (G(x_2^m) ∩ H(y_2^m))= Q(x_1, y_1) x Q(x_2 , y_2)
(FSM2) Q(r(x_1,y_1))^m = G ($x_1 y_1$)^m x H ($x_1 y_1$)^m
$\dot\supseteq$(G(x_1^m) xH(y_1^m)) = Q(x_1, y_1)
Therefore G_{S1}x H_{S2} = Q_{S1}x $_{S2}$ < M_1 x M_2.

Proposition 2.9: If $G_S < M$, then $G(0^m{}_M) \dot{\supseteq} G(x^m)$ for all $x \in S$.

Proof: Since (G ,S) is not a fuzzy soft sub module of M, then for all $x \in S$,
$G(0^m{}_M) = G(x^m - x^m) \dot{\supseteq} G(x^m) \cap G(x^m) = G(x^m)$ for all $x \in S$.

Proposition 2.10: If $Gs < M$ and $G(1_mM) = G(0^mM)$, then $S_G = \{x \in S \,/\, G(x^m) = G(0^mM)\}$ is a fuzzy sub module of S.

Proof : we need to show that $0^m{}_M \in S_G$, $1_{mM} \in S_G$, $x^m\text{-}y^m \in S_G$ $(y \neq 0_M)$ for all $x,y \in S_G$, which means that (i) $G(0_M{}^m) = G(0_M{}^m)$, (ii) $G(1_M{}^m) = G(0_M{}^m)$ and (iii) $G(x^m\text{-}y^m) = G(0_M{}^m)$ and (iv) $G(rx)^m = G(0_M{}^m)$ have to be satisfied. (i) is obivious and (ii) comes from the assumption. Since $x,y \in S_G$, then $G(x) = G(y) = G(0_M{}^m)$. Since (G, S) is a sub module of M, then $G(x^m\text{-}y^m) \dot{\supseteq} G(x^m) \cap G(y^m) = G(0_M{}^m)$ and $G(rx)^m \dot{\supseteq} G(x^m) = G(0_M{}^m)$ for all $x,y \in S_G$ $(y \neq 0_M)$. Moreover, by the proposition 2.9, $G(0_M{}^m) \dot{\supseteq} G(x^m\text{-}y^m)$ and $G(0_M{}^m) \dot{\supseteq} G(rx)^m$. Therefore S_G is a fuzzy soft sub module of S.

Definition-2.11:Let (G , S) be a fuzzy soft sub module of M. Then

(i) (G, S) is said to be trivial if $G(x^m) = \{0^mM\}$ for all $x \in S$.

(ii) (G, S) is said to be whole if $G(x^m) = M$ for all $x \in S$.

Proposition 2.12: Let (G ,S_1) and (H, S_2) be fuzzy soft sub modules of M. Then

(i) If (G , S_1) and (H , S_2) are trivial fuzzy soft sub modules of M, then (G, S_1) $\Cap$ (H , S_2) is a trivial fuzzy soft sub module of M.

(ii) If (G , S_1) and (H , S_2) are whole fuzzy soft sub modules of M, then (G, S_1) $\Cap$ (H , S_2) is a whole fuzzy soft sub module of M.

(iii) If (G , S_1) is a trivial fuzzy sub module of M and (H , S_2) is a whole fuzzy soft sub module of M, then (G, S_1) $\Cap$ (H , S_2) is a trivial fuzzy soft sub module of M.

Proof: The proof is easily seen by definition- 2.1and definition-2.11 and theorem -2.6.

Theorem-2.13: Let M_1 and M_2 be modules and $(G_1, S_1) < M_1$, $(G_2, S_2) < M_2$. If $f : S_1 \rightarrow S_2$ is a module homomorphism, then

(a) If f is epimorphism, then $(G_1, f^{-1}(S_2)) < M_1$,

(b) $(G_2, f(S_1)) < M_2$,

(c) $(G_1, \mathrm{Ker}\, f) < M_1$.

Proof: (a) Since $S_1 < M_1$, $S_2 < M_2$ and $f : M_1 \rightarrow M_2$ is a module epimorphism, then it is obivious that $f^{-1}(S_2) < M_1$. Since $(G_1, S_1) < M_1$ and $f^{-1}(S_2) \subseteq S_1$, $G_1(x^m\text{-}y^m) \dot{\supseteq} \min\{G_1(x^m), G_1(y^m)\}$
for all $x,y \in f^{-1}(S_2)$ and $G_1(rx)^m \dot{\supseteq} G_1(x^m)$.
Hence $(G_1, f^{-1}(S_2)) < M_1$.

(b) Since $S_1 < M_1$, $S_2 < M_2$ and $f : S_1 \rightarrow S_2$ is a module homomorphism, then $f(S_1) < S_2$.Since $f(S_1) < S_2$, the result is obivious by definition -2.7.

(c) By theorem 2.13 (a), $(G_1, \mathrm{Ker}\, f) < M_1$. Then $(G_2, \mathrm{Ker}\, f) = (G_2, \{0s_2\})) < M_2$ by theorem 2.13(b).

Corollary2.14: Let $(G_1, S_1) < M_1$, $(G_2,S_2) < M_2$ and $f: S_1 \rightarrow S_2$ is a module homomorphism. Then $(G_2, \{0s_2\})) < M_2$.

Proof: By theorem 2.13 (c) ,$(G_1, \mathrm{Ker}\, f) < M_1$. Then $(G_2, \mathrm{Ker}\, f) = (G_2, \{0s_2\})) < M_2$ by theorem 2.13(b).

Definition-2.15: Let S be a soft subfield of M and let (G, S) be a soft set over M. If for all $x,y \in S$,

(FS₣1) $G(x^m - y^m) \dot{\supseteq} G(x^m) \cap G(y^m) = \min\{G(x^m), G(y^m)\}$,

(FS₣2) $G(xy^{-1})^m \dot{\supseteq} G(x^m) \cap G(y^m) = \min\{G(x^m), G(y^m)\}$ $(y \neq 0_M)$, then the soft set (G, S) is called a fuzzy soft subfield of M and denoted by (G , S) < M or simply $G_S < M$.

Example 2.16: Let $M = (Z_3 , + , \circ)$, $S = Z_3 < Z_3$ and the soft set (G, S) over ₣, where $G : S \rightarrow P(M)$ is a set- valued function by $G(0) = Z_3$, $G(1) = G(2) = \{1,2\}$. Then it can be easily seen that (G ,S) < M.

However if we define the soft set (H ,S) over ₣ such that $H : S \rightarrow P(M)$ is a set-valued function defined by $H(0) = Z_3$, $H(1) = \{1,2\}$ and $H(2) = \{0,1\}$, then $H(2.2^{-1}) = H(2.2) = H(1) = \{1,2\} \not\supseteq \min\{H(2), H(2)\} = H(2) = \{0,1\}$. It follows that (H ,S) is not a fuzzy soft subfield of M.

Definition 2.17: Let I be an ideal of R and let (₣ , I) be a soft set over R. If for all $x,y \in I$ and $r \in R$,

(FSI1) ₣ $(x^m - y^m) \dot{\supseteq} \min\{$₣$(x^m)$, ₣$(y^m)\}$ = ₣$(x^m) \cap$ ₣(y^m)

(FSI2) ₣ $(rx)^m \dot{\supseteq}$ ₣(x^m)

(FSI3) ₣ $(xr)^m \dot{\supseteq}$ ₣(x^m), then (₣ , I) is called a fuzzy soft ideal of R and denoted by ₣$_I < R$.

Example 2.18: Let $R = (Z_{12} , + , \circ)$, $I_1 = \{0,6\} < R$ and the soft set (₣ , I_1) over R , where ₣ : $I_1 \rightarrow P(R)$ is a set-valued function defined by ₣$(0) = Z_{12}$ and ₣$(6) = \{1,7\}$. It is denoted as ₣$_{I1} < R$.Also let $I_2 = \{0,4,8\} < R$ and the soft set (G , I_2) over R , where $G : I_2 \rightarrow P(R)$ is a set-valued function defined by $G(0) = Z_{12}$, $G(4) = G(8) = \{3,9\}$. It is denoted as $G_{I2} < R$.

However if we define the soft set (H ,I_2) over R such that the soft set $H : I_2 \rightarrow P($₣$)$ is a set-valued function defined by $H(0) = Z_{12}$, $H(4) = \{1,3\}$ and $H(8) = \{1,2\}$, $\not\supseteq H(4) = \{1,3\}$. It follows that (H , I_2) is not a fuzzy soft subideal of R.

III. OPERATIONS ON SUBFIELDS STRUCTURES OVER A FIELD

In this section, we study some operations on subfields over a field.

Theorem3.1: The intersection of two fuzzy soft subfields of ₣ is also a fuzzy soft subfield of ₣.

Proof: Since S_1 and S_2 are fuzzy subfields of ₣, then $S_1 \cap S_2$ is a fuzzy subfield of ₣.

By definition 2.15 , let
$G_{S1} \cap H_{S2} = (G,S_1) \cap (H,S_2) = (T , S_1 \cap S_2)$, where $T(x^m) = G(x^m) \cap H(y^m)$ for all $x \in S_1 \cap S_2 \neq \phi$, then for all $x,y \in S_1 \cap S_2$.

(FS₣1) $T(x^m - y^m) = G(x^m - y^m) \cap H(x^m\text{-}y^m)$

$\dot{\supseteq} (G(x^m) \cap G(y^m)) \cap (H(x^m) \cap H(y^m))$

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$= (G(x^m) \cap H(x^m)) \cap (G(y^m) \cap H(y^m))$
$= T(x^m) \cap T(y^m)$,
(FSⱣ2) $T(xy^{-1})^m = G(xy^{-1})^m \cap H(xy^{-1})^m \supseteq (G(x^m) \cap G(y^m)) \cap (H(x^m) \cap H(y^m)) = (G(x^m) \cap H(x^m)) \cap (G(y^m) \cap H(y^m)) = T(x^m) \cap T(y^m)$. Therefore $G_{S1} \cap H_{S2} = T_{S1 \cap S2} <$ Ⱡ.

Definition 3.2: Let $Ⱡ_1$ and $Ⱡ_2$ be fields and let (G , S_1) and (H, S_2) be two fuzzy soft subfields of $Ⱡ_1$ and $Ⱡ_2$ respectively. The product of fuzzy soft subfields (G, S_1) and (H , S_2) is defined as (G , S_1) x (H , S_2) = (Q ,$S_1 x S_2$), where Q(x,y) = Ⱡ(x) x G(y) for all (x,y) ∊ $S_1 x S_2$.

Theorem 3.3 : The product of two fuzzy soft subfields of Ⱡ is also a fuzzy soft subfield of Ⱡ.

Proof: Since S_1 and S_2 are soft subfields of Ⱡ, then $S_1 \cap S_2$ is a soft subfield of Ⱡ.

By definition 2.15 , let $G_{S1} x H_{S2}$ = (G,S_1) x (H,S_2) = (Q , $S_1 x S_2$),where Q(x,y) = Ⱡ(x) x G(y) for all (x,y) ∊ $S_1 x S_2$.Then for all (x_1,y_1), (x_2, y_2) ∊ S_1 x S_2.

(FSⱣ1) $Q((x_1, y_1)^m – (x_2, y_2)^m) = Q (x_1^m-x_2^m, y_1^m-y_2^m)$
$= G(x_1^m-x_2^m) \times H(y_1^m-y_2^m) \supseteq (G(x_1^m) \cap G(x_2^m)) \times (H(y_1^m) \cap H(y_2^m))$
$= (G(x_1^m) \cap H(y_1^m)) \times (G(x_2^m) \cap H(y_2^m))$
$= Q(x_1, y_1) \times Q(x_2 , y_2)$
(FSⱣ2) $Q((x_1,y_1) (x_2, y_2)^{-1})^m = Q((x_1x_2^{-1})^m (y_1y_2^{-1})^m)$
$= G (x_1x_2^{-1})^m \times H (y_1y_2^{-1})^m \supseteq (G(x_1^m) \cap G(x_2^m)) \times (H(y_1^m) \cap H(y_2^m))= (G(x_1^m) \cap H(y_1^m)) \times (G(x_2^m) \cap H(y_2^m))= Q(x_1, y_1) \times Q(x_2 , y_2)$

Therefore $G_{S1} x H_{S2} = Q_{S1} x_{S2} < Ⱡ_1$ x $Ⱡ_2$.

Proposition 3.4: If G_S < Ⱡ, then $G(0^m{}_{Ⱡ}) \supseteq G(x^m)$ for all x ∊ S.

Proof: Since (G ,S) is not a fuzzy soft subfield of Ⱡ, then for all x ∊ S, $G(0^m{}_{Ⱡ}) = G (x^m – x^m) \supseteq G(x^m) \cap G(x^m) = G(x^m)$ for all x ∊ S.

***Proposition 3.5*:** If Gs < Ⱡ and $G(1^m Ⱡ) = G(0^m Ⱡ)$, then S_G = {x ∊ S / $G(x^m) = G(0^m Ⱡ)$} is a fuzzy subfield of S.

Proof : we need to show that $0^m{}_{Ⱡ} \in S_G$, $1^m{}_{Ⱡ} \in S_G$, $x^m-y^m \in S_G$ (y ≠ $0_Ⱡ$) for all x,y∊ S_G, which means that (i) $G(0_Ⱡ{}^m) = G(0_Ⱡ{}^m)$, (ii) $G(1_Ⱡ{}^m) = G(0_Ⱡ{}^m)$, (iii) $G(x^m-y^m) = G(0_Ⱡ{}^m)$ and (iv) $G(xy^{-1})^m = G(0_Ⱡ{}^m)$ have to be satisfied. (i) is obivious and (ii) comes from the assumption. Since x,y∊ S_G, then G(x) = G(y) = $G(0_Ⱡ{}^m)$. Since (G, S) is a subfield of Ⱡ, then $G(x^m-y^m) \supseteq G(x^m) \cap G(y^m) = G(0_Ⱡ{}^m)$ and $G(xy^{-1})^m \supseteq G(x^m) \cap G(y^m) = G(0_Ⱡ{}^m)$ for all x,y∊ S_G (y ≠ $0_Ⱡ$). Moreover, by the proposition- 3.4, $G(0_Ⱡ{}^m) \supseteq G(x^m-y^m)$ and $G(0_Ⱡ{}^m) \supseteq G(xy^{-1})^m$. Therefore S_G is a fuzzy subfield of S.

Definition 3.6 **:** Let (G , S) be a fuzzy soft subfield of Ⱡ. Then

(i) (G, S) is said to be trivial if $G(x^m) = \{0^m Ⱡ\}$ for all x ∊ S.

(ii) (G, S) is said to be whole if $G(x^m)$ = Ⱡ for all x ∊ S.

Proposition 3.7: Let (G ,S_1) and (H, S_2) be fuzzy soft subfields of Ⱡ. Then

(i) If (G , S_1) and (H , S_2) are trivial fuzzy soft subfields of Ⱡ, then (G, S_1) ⋒ (H , S_2) is a trivial fuzzy soft subfield of Ⱡ.

(ii) If (G , S_1) and (H , S_2) are whole fuzzy soft subfields of Ⱡ, then (G, S_1) ⋒ (H , S_2) is a whole fuzzy soft subfield of Ⱡ.

(iii) If (G , S_1) is a trivial fuzzy soft subfield of Ⱡ and (H , S_2) is a whole fuzzy soft subfield of Ⱡ, then (G, S_1) ⋒ (H , S_2) is a trivial fuzzy soft subfield of Ⱡ.

Proof: The proof is easily seen by definition 2.15and definition-3.6 and theorem -3.1.

Theorem 3.8: Let $Ⱡ_1$ and $Ⱡ_2$ be fields and (G_1, S_1) < $Ⱡ_1$, (G_2, S_2) < $Ⱡ_2$. If f : $S_1 \to S_2$ is a field homomorphism, then

(a) If f is epimorphism, then (G_1, $f^{-1}(S_2)$) < $Ⱡ_1$,
(b) (G_2, f(S_1)) < $Ⱡ_2$,
(c) (G_1, Ker f) < $Ⱡ_1$.

Proof: (a) Since S_1< $Ⱡ_1$, S_2 < $Ⱡ_2$ and f :$Ⱡ_1 \to Ⱡ_2$ is a field epimorphism, then it is obivious that $f^{-1}(S_2)$ < $Ⱡ_1$. Since (G_1, S_1) < $Ⱡ_1$ and $f^{-1}(S_2) \subseteq S_1$, $G_1(x^m-y^m) \supseteq \min \{ G_1(x^m) , G_1(y^m)\}$ for all x,y∊ $f^{-1}(S_2)$ and $G_1(xy^{-1})^m \supseteq \min \{ G_1(x^m) , G_1(y^m)\}(y \neq 0_{Ⱡ1})$.

Hence (G_1, $f^{-1}(S_2)$) < $Ⱡ_1$.

(b) Since S_1< $Ⱡ_1$, S_2 < $Ⱡ_2$ and f : $S_1 \to S_2$ is a field homomorphism, then f(S_1) < S_2.Since f(S_1) < S_2, the result is obvious by definition 3.2.

(c) By theorem 3.8 (a), (G_1, Ker f) < $Ⱡ_1$. Then (G_2, Ker f) = (G_2, {$0s_2$})) < $Ⱡ_2$ by theorem 3.8(b).

Corollary 3.9 **:** Let (G_1, S_1) < $Ⱡ_1$, (G_2,S_2) < $Ⱡ_2$ and f: $S_1 \to S_2$ is a field homomorphism, then (G_2, {$0s_2$})) < $Ⱡ_2$.

Proof: By theorem 3.8 (c) ,(G_1, Ker f) < $Ⱡ_1$ .Then (G_2, Ker f) = (G_2, {$0s_2$})) < $Ⱡ_2$ by theorem 3.8

IV. CORRELATION COEFFICIENT'S BETWEEN INTERVAL VALUED FUZZY SOFT SUBFIELD (A) AND FUZZY SOFT SUB MODULE (B)

***Definition 4.1*:** Let A and B are two IVFS . The correlation between A and B is defined as CORR (A,B) = ∑AB divides ∑A and ∑B.

Example 4.2: Let A = [0.2 , 0.3] and B = [0.4 , 0.5] be two IVFS. Then ∑AB = 0.23, ∑A = 0.6 and ∑B = 0.8. Hence CORR (A,B) = 0.31.

In general, when i = 1,2,……..,n , correlation coefficient occurs whose value in [0 , 1] .

Definition 4.3: Let A = [0.2 , 0.5] and B = [0.5 , 0.8] be two IVFS.

Union **:** A U B = max {A, B } = 0.8

***Intersection*:** A ∩ B = min {A , B } = 0.2.

Note 4.4: The actual correlation score value between two IVFS's A and B is given by

Score value ʋ= min max{ A U B, A∩B}.

From the definition 4.3, score value is obtained by ʋ = 0.2.

Remark 4.5: CORR (A) ≤ CORR(B)
CORR(AB) ≤ CORR (A)

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Score CORR(A) ≤ Score CORR(AB).
In n^{th} occurrence of correlation is $\rho = \sum_{i=1}^{n}\sum_{i=1}^{n} AiBi$ divides $\sum_{i=1}^{n} Ai2 \sum_{i=1}^{n} Bi2$
Now if x ∈ CORR(A_n) and y ∈ CORR(A_n) such that x ∧ y ∈ CORR(A_n).
If x ∉CORR(A_n) and y ∉ CORR(A_n) , then the following four cases arise.
Case-1: x ∈ X / A_n and y ∈ X /A_n
Case-2: x ∈ X / A_{n-1} and y ∈ X /A_{n-1}
Case-3- x ∈ X / A_n and y ∈ X /A_{n-1}
Case-4: x ∈ A_{n-1} and y ∈ R /A_n.

The correlation measure is obtained for K and L
$\rho = C(K, L) / \sqrt{C(K,K)\,C(L,L)}$
Let {Ai / i = 0,1,2,........., k } be a family of IVFS such that
(i) CORR(A_n) ⊆ CORR (A_1) ⊆ CORR (A_2) .... ⊆ CORR(A_k) = X.
(ii) $A_n{}^* = A_n / A_{n-1}$ /........./ A_{-1}.

In this section, we form a new algorithm to find out suitable correlations between two interval-valued fuzzy sets.

V. ALGORITHM

Step-1: Set an IVFS according the problem.
Step-2: Preparation of table for calculations.
Step-3: find out AUB and A∩B.
Step-4: Correlation coefficient CORR (A,B) = ∑AB divides ∑A and ∑B.
Step-5: Actual score value using min max {AUB, A∩B}.
Step-6: Compare the result.

4.7 Problem**:** For an interval valued fuzzy sets A and B, supposed to be 5 observations, the correlation coefficient has been formulated according to the following IVFS sets A and B as
X = { A / [0.2, 0.4],[0.3, 0.9],[0.1, 0.4],[0.2, 0.7],[0.8, 0.9]and
B / [0.3, 0.6], [0.8, 0.4],[0.2, 0.7] ,[0.4, 0.8],[0.7, 0.4]}.

Step-2: Preparation of table for calculations

N	Field A	Field B
1	[0.2, 0.4]	[0.3, 0.6]
2	[0.3, 0.9]	[0.8, 0.4]
3	[0.1, 0.4]	[0.2, 0.7]
4	[0.2, 0.7]	[0.4, 0.8]
5	[0.8, 0.9]	[0.7, 0.4]

Step-3: find out AUB and A∩B.
In this problem, we have to find union and intersection can be calculated for 5 observations as
AUB = { [0.3, 0.6], [0.8. 0.9], [0.2, 0.7],[0.4, 0.8],[0.8, 0.9] }
A∩B = { [0.2, 0.4], [0.3, 0.4], [0.1, 0.4], [0.2, 0.7], [0.7, 0.4] }.

Step-4: Correlation coefficient CORR (A,B)
Using step-3, by routine calculations, we can calculate ρ = 0.032.
Step-5: by using Note-4.4, the Actual score value is calculated as υ = [0.1, 0.1] =0.1.
Step-6: Comparing step-4 and step-5, all values lies in the interval [0, 1].

VI. CONCLUSION

In this work, we discuss the analysis of fuzzy soft sub modules over subfields of a field. Some related properties about algebraic substructures of soft fields and soft sub modules are investigated and illustrated by many examples. we studied certain concept and derivations in which a new kind of interval-valued fuzzy correlation method. Also we compare the actual preparation value with calculation value in a simple manner. One can obtain squared value of IVFS by using the above algorithm.

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