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Appraisal of the Sum of the Polygonal Numbers of Even Order from Stella Octangula Number and Pronic Number Using the Division Algorithm

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***Abstract*—**We Present the sum of the Polygonal numbers of even order are derived from Stella Octangula number and Pronic number using division algorithm.

Keywords—Polygonal numbers, The division algorithm, Stella Octangula number and Pronic number.

I. INTRODUCTION

The theory of numbers has always occupied a unique position in the world of Mathematics. It is a kind of general theory concerning the notion of number and its generalization starting from integers.

In [1-4], theory of numbers were discussed. In [5], a function $A: N \to N$ is defined by $A(n) = m$ where m is the smallest natural number such that n divides $m + \sum_{1}^{n} k^2$ & in [6], a function $A(n)$ is given by $A(n) = m$ where k is the smallest natural number such that n divides $m + \sum_{1}^{n} m$ and $m + \sum_{1}^{n} k^3$ were discussed. In [7&8] Stella Octangula number and Pronic number were evaluated using z-transform & Pronic number was analyzed for its special dio-triples. Recently, in [9&10] centered polygonal numbers and polygonal numbers were evaluated using division algorithm. In this communication, we determine a function $A(n)$ given by $A(n) = m$ where m is the sum of the Polygonal numbers of even sides, such that SO_n and Pro_n divides $m + (\text{a polynomial})$.

Section I contains the introduction of the division algorithm, Section II contain the notations, Section III explain the methodology of determining the sum of the polygonal from Stella Octangula number and Pronic number using division algorithm, Section IV concludes research work with future directions.

II. NOTATIONS

SO_n = Stella Octangula number of rank 'n'.

Pro_n = Pronic number of rank 'n'.

$T_{m,n}$ = Polygonal number with sides 'm' and rank 'n'.

III. METHOD OF ANALYSIS

SECTION A:

Determination of the sum of the Polygonal numbers of even sides from Stella Octangula number:

Let $A: N \to N$ be defined by $A(n) = m$ where m is the smallest natural number such that SO_n divides $m + \left(2n^3 - (4k-1)n^2 + (4k-4)n\right)$. If SO_n divides $2n^3 - (4k-1)n^2 + (4k-4)n$, then $A(n) = (4k-1)n^2 - (4k-3)n$, otherwise $A(n) = \left((4k-1)n^2 + (4k-3)n\right) - r$, where r is the smallest non-negative remainder when $2n^3 - (4k-1)n^2 + (4k-4)n$ is divided by SO_n. Hence A is defined for all n. By division algorithm, such remainder is given by $\left(2n^3 - (4k-1)n^2 + (4k-4)n\right) - qSO_n$ where q is the quotient when $2n^3 - (4k-1)n^2 + (4k-4)n$ is divided by SO_n & is given by the greatest integer function of $\frac{\left(2n^3 - (4k-1)n^2 + (4k-4)n\right)}{SO_n}$,

$$\text{i.e., } q = \frac{\left(2n^3 - (4k-1)n^2 + (4k-4)n\right)}{SO_n}$$

So that,

$$A(n)=(2n^3-n)-\left\{\left(2n^3-(4k-1)n^2+(4k-4)n\right)-\left[\frac{\left(2n^3-(4k-1)n^2+(4k-4)n\right)}{2n^3-n}\right]\times 2n^3-n\right\}$$

$$=2n^3-n-2n^3+(4k-1)n^2-(4k-4)n \quad +\left[\frac{2n^3-n}{2n^3-n}-\frac{(4k-1)n^2-(4k-3)n}{2n^3-n}\right]\times 2n^3-n$$

$$=(4k-1)n^2-(4k-3)n+\left[1-\frac{(4k-1)n^2-(4k-3)n}{2n^3-n}\right]\times 2n^3-n$$

$$\therefore \quad A(n)=\left((4k-1)n^2-(4k-3)n\right)+0 \qquad \left(\text{Since } 0<\frac{(4k-1)n^2-(4k-3)n}{2n^3-n}<1\right)$$

$$=\left((4k-1)n^2-(4k-3)n\right)$$

$$=(2k-1)n^2-(2k-2)n+2kn^2-(2k-1)n$$

$A(n)=$ Sum of the Polygonal numbers of even sides.

SECTION B:

Determination of the sum of the Polygonal numbers of even sides from Pronic number:

Let $A:N\to N$ be defined by $A(n)=m$ where m is the smallest natural number such that Pro_n divides $m+\left((-4k+2)n^2+(4k-2)n\right)$. If Pro_n divides $(-4k+2)n^2+(4k-2)n$, then $A(n)=(4k-1)n^2-(4k-3)n$, otherwise $A(n)=\left((4k-1)n^2-(4k-3)n\right)-r$, where r is the smallest non-negative remainder when $(-4k+2)n^2+(4k-2)n$ is divided by Pro_n. Hence A is defined for all n. By division algorithm, such remainder is given by $((-4k+2)n^2+(4k-2)n)-q\text{Pro}_n$ where q is the quotient when $(-4k+2)n^2+(4k-2)n$ is divided by Pro_n & is given by the greatest integer function of $\frac{\left((-4k+2)n^2+(4k-2)n\right)}{\text{Pro}_n}$,

$$\text{i.e., } q=\frac{\left((-4k+2)n^2+(4k-2)n\right)}{\text{Pro}_n}$$

So that

$$A(n)=n^2+n-\left\{(-4k+2)n^2+(4k-2)n-\left[\frac{(-4k+2)n^2+(4k-2)n}{n^2+n}\right]\times n^2+n\right\}$$

$$=n^2+n+4kn^2-2n^2-4kn+2n+\left[\frac{n^2+n}{n^2+n}-\frac{(4k-1)n^2-(4k-3)n}{n^2+n}\right]\times n^2+n$$

$$=(4k-1)n^2-(4k-3)n+\left[1-\frac{(4k-1)n^2-(4k-3)n}{2n^3-n}\right]\times n^2+n$$

$$\therefore \quad A(n)=\left((4k-1)n^2-(4k-3)n\right)+0 \qquad \left(\text{Since } 0<\frac{(4k-1)n^2-(4k-3)n}{n^2+n}<1\right)$$

$$=\left((4k-1)n^2-(4k-3)n\right)$$

$$=(2k-1)n^2-(2k-2)n+2kn^2-(2k-1)n$$

$A(n)=$ Sum of the Polygonal numbers of even sides.

From Section A & Section B, We attain, *A(n)* is the Sum of the Polygonal numbers of even sides for different values of k where $k=1,2,\ldots 7$ is given in the table below.

Table 1. Examples

n	*A(n)*	Sum of the Polygonal numbers of even sides
1	$3n^2-n$	$T_{4,n}+T_{6,n}$
2	$7n^2-5n$	$T_{8,n}+T_{10,n}$
3	$11n^2-9n$	$T_{12,n}+T_{14,n}$
4	$15n^2-13n$	$T_{16,n}+T_{18,n}$
5	$19n^2-17n$	$T_{20,n}+T_{22,n}$
6	$23n^2-21n$	$T_{24,n}+T_{26,n}$
7	$27n^2-25n$	$T_{28,n}+T_{30,n}$

IV. CONCLUSION

To conclude that, one may find the other special numbers using division algorithm.

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