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Research Paper

Magnetohydrodynamic (MHD) nanofluid flow over a non- linear stretchable surface in the presence of Heat generation/ absorption

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Abstract— The emphasis of this research is on the effects of heat generation/absorption on MHD nanofluid flow over a stretchable surface. The influence of Prandtl number, solar radiation, as well as other physical parameters are all taken into account in relation to the heat generation/absorption. The boundary layer approximation alongside similarity transformation are used to turn the governing system of partial differential equations into an ordinary differential system, which is then solved numerically using the Runge- Kutta- Fehlberg method and the Shooting approach. The focus of this study is on the effects of heat generation/absorption on MHD nanofluid flow with a two-dimensional stagnation point along a stretchy surface. Magnetic fields, sun radiation, and other physical characteristics all have an impact. Graphs are used to study and assess the consequences of various physical characteristics. Temperature, nanoparticle concentration and Nusselt number are all rapidly decreasing functions, according to the information. When the heat generation/absorption parameter is continued to increase, the momentum fluctuates.

Keywords— Boundary layer approximation, Solar Radiation, Nanofluid, Heat Generation, heat Absorption, Stretching Surface, MHD.

1. Introduction

Throughout earlier civilizations, humanity collected solar energy, or the light refracting and heat of the sun, by using a variety of quick machines at a variety of speeds, Nasrin et al. [1]. Magnetohydrodynamics (MHD) is a mixture of fluid dynamics and electromagnetism, i.e. the effect of a magnetic field on the properties or management of an electric conductivity of a fluid. Because of its importance in a wide range of research and production applications, such as electromagnetic blenders, MHD power generators, plasma education, nuclear reactors, the petroleum sector of the economy, and aerodynamic boundary layer control, chemical processes. A vast number of research have lately been conducted on MHD fluid flow among which are Harada et al. [2], Shang et al. [3] and Abricka et al. [4]. In past few years, the research for a superior technique that boost the heat exchange rates are currently possibilities seems to have been a key priority in the manufacturing industry. Heat-conducting materials fall short of industrialist aspirations, making energy transmission problematic. And according to study, suspending nanoparticles with a size of 100 nm is the best way to improve thermal conductivity. Benefits of nanofluids are essential in many domains, such as nuclear power, delivery, air conditioning, cooling, and ventilation. Several investigations have been conducted on nanofluids and how they affect flow, as well as the range of uses and possibilities they have. These Incompressible viscous nanoparticles are kept together by bending slip restrictions. Prasannakumara et al. [5] examined the use of partial derivatives to study the breakdown of viscous and nanofluid interactions in governing equations. There are a lot of agreement when compared to the current outcomes. Anisotropic slip on a porous stretching/shrinking plate caused a halted flow of ferromagnetic nanofluid, as described by Nadeem et al. [6]. Madhu et al. [7] established non-Newtonian flow behavior for such Maxwell controlling flow model. The governing flux of unsteady MHD is calculated using the finite element model on the geometrically extending surface. Palani et al. [8] reported a significantly higher Chemical changes within an UCM fluid involving this turbulence. The effects of a variety of dimensionless factors flow were studied. Choi [9] first termed mitigating nanoparticles for their contact boosted thermophysical characteristics. Buongiorno et al. [10] studied the temperature distribution in the convection medium for nanofluids. Vajravelu et al. [11] explored magnetohydrodynamic heat sources, including thermal radiation, on several types of sheets. Non-Newtonian controlling movement, as well as linear order velocity with slip effects, were defined by in there. A few important researches on these fluids include Kakaç et al. [12], Nawaz et al. [13]. As a consequence, numerous methods for improving the heat in fact, methods for increasing aversion of such substances by adding nanoparticles to fluids were developed. Because nanometer-sized materials offer exceptional mechanical properties, nanotechnology is frequently utilized in commercial processes. Choi et al. [14] and [15] revealed that a little amount of nanoparticles (less than 1% by volume) increased the thermal conductivity of typical heat exchange liquids by about two times. Khanafer et al. [16] appear to be the most recent researchers to look at the heat transmission characteristics of nanofluids within the enclosure while taking nanoparticle dispersion into account. Based on the evidence, many people believe it is predicted that one of the key elements driving this century's upcoming significant economic revolution will be nanotechnology. The researchers employed the most basic border conditions possible, such as maintaining a constant temperature and nanoparticle proportion along the boundary. The Minkowycz et al. [17], discussed on the problem of spontaneous convection across a vertical plate in a porous material saturated with a nanofluid, which was further investigated by Nield and Kuznetsov [18]. The nanofluid idea accounts for Brownian motion and thermophoresis properties and the porous structure was simulated using the Darcy model.

Ghasemi et al. [19] used the probative quadrature method to examine the impact of a nanofluid further than a stretchable sphere on the magnetization. Wubshet Ibrahim et al. [20] looked at how a magnetic field affected flow at the stagnation point and thermal expansion from a nanofluid to an expanding sheet. To solve the governing equations numerically, researchers employed the fourth-order Runge-Kutta method with the firing approach. Wakif et al. [21] offered a novel simple mechanism for resolving the issue. Khader et al. [22] studied the thermally stability of a biphasic heterogeneous hybridized nanofluid with corresponding volumetric percentages of Al_2O_3 and CuO nanoparticles in the aqueous solution in a confined area. Thumma et al. [23] looked at the convective MHD nanofluids boundary-layer continuous movement of beyond just a nonlinearly pitched stretching / shrinking sheet while taking viscous dissipation into account. The method of differential Transformation was applied by Ghasemi et al. [24] to investigate the DTM (heat generation and thermal conductivity).

This research is targeted in applying the Runge- Kutta-Fehlberg technique and indeed the shooting technique to proactively address the MHD boundary layer flow of nanofluids framework along a stretchable surface in the presence of heat generation/absorption, and hence, to compare the conclusions reached with that of Ghasemi et al. [25]. Who examined the influence of solar radiation on the flow of nanofluids across stretchable surface in the presence of heat generation/ absorption. Multiple key parameters, including the radiation parameter, magnetic parameters, thermophoresis, Brownian motion, and Prandtl number, are also investigated for their impact on momentum, Nusselt number, nanoparticle concentration and temperature profiles. This very same study extends, as according to Nageeb et al. [26] and Mushtaq et al. [27], has had many conceivable designs and implementations through processing technologies, fiber glass manufacturing, and metallic materials procedures which encourage cooling of long ribbons or natural fiber other than attempting to draw them through some liquids, strengthening as well as wave soldering of copper wires, as well as other mechanisms in which the attributes of the finished article emerge to be highly dependent on it.

2. Mathematical Model

In the presence of heat generation/absorption through radiation from the sun, an uninterrupted two-dimensional nanofluid boundary layer flow with the linear velocity of $u_w(x) = ax$, whereby a is constant, is examined. As shown in Figure1. A magnetic field of uniform intensity is considered to be present.

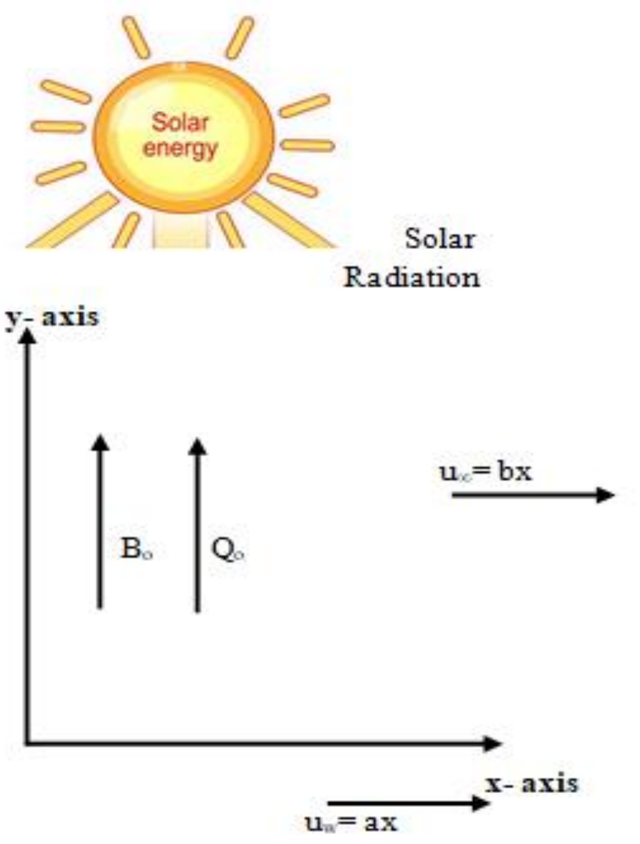


Figure 1 demonstration of the real model.

The force is proportional but also opposing from the source in the orientations in which the surface is extended on the edge and at right- angle of x and y– axes respectively. At the coating's surface, the fluid flow is kept steady. The temperature of the sheet's surface is T_w, free stream flow of the fluid is $u_\infty(x)$, T is the temperature of the surrounding fluid is, M is then magnetic field, C is the nanoparticle concentration. Let us consider an external electrical field to be zero and that charge polarization-induced is negligible. Following that, a mass and heat transfer analysis has been performed, taking into consideration the effects of heat generation/absorption together with supplementary relevant properties. But during radiative heating procedure, the material surface temperature T_w coincides towards the passive fluid temperature T_f. The nanoparticle concentration at the wall is denoted by C_w, and ambient concentration is denoted by C_∞.

Similar to those used by Ghasemi et al. [25], the equations below describe the fluid flow for (Momentum, Temperature, and Nanoparticle Concentration) in the presence of Heat generation/absorption:

$$\frac{\partial u}{\partial y}+\frac{\partial u}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{du_\infty}{\partial x}+v_f\frac{\partial^2 u}{\partial x^2}-\frac{\sigma_e B_0^2}{\rho_f}(u-u_\infty) \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{v_f}{C_f}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{(\rho C)_f}\left(\frac{\partial q_r}{\partial y}\right)$$
$$+\frac{\sigma_e B_0^2}{(\rho C)_f}(u_\infty-u)^2+\frac{Q}{\rho_{nf}}(T-T_\infty)$$
$$+\tau D_B\left[\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \quad (3)$$

$$u\frac{\partial C}{\partial y}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \quad (4)$$

In which kinematic viscosity is denoted by v_f, fluid's electrical conductivity is denoted by σ_e, and the magnetic field intensity is denoted by B_0, $\tau=\frac{(\rho C)_p}{(\rho C)_f}$, refers to the ratio of the effective heat capacity of nanoparticles to the heat capacity of the base fluid, whereby q_r signifies the quantity of radiative heat flux, u as well as v signify components of velocity in the x- and y-axes, respectively. [Raptis et al. (1998), Brewster et al. (1972), as well as Sparrow and Sparrow (1978), Rosseland approximation can be applied to estimate q_r using the thermal radiation:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}=\frac{16\sigma^*}{3k^*}T^3\frac{\partial T}{\partial y} \quad (5)$$

The Stefan–Boltzman constant is σ^* while k^* is the average absorption coefficient. The radiative heat flow rate analysis is applied on the non- linear Rosseland approximation. As a consequence, the boundary conditions for convective heat transfer can be expressed as

$$at\ y=0:-k\frac{\partial T}{\partial y}=h(T-T_f),\ C=C_w$$
$$at\ y\to\infty: T\to T_{\infty,}\ C\to C_\infty \quad (6)$$

These are the dimensionless quantities as by Ghasemi et al. (2021)

$$\eta=\sqrt{\frac{a}{v_f}}y,\ u=axf'(\eta),\ v=-\sqrt{av_f}\,f(\eta) \quad (7)$$

For Eq. (2), that the very first component on the right hand side would be transposed to $\alpha\frac{\partial}{\partial y}\left[\frac{\partial T}{\partial y}\left(1+R_d\left(\theta_w-1\right)\theta^3\right)\right]$, when $R_d=16\sigma^* T_\infty^3/3kk^*$ this same non-dimensional temperature gradient is defined simply $\theta(\eta)=T-T_\infty/T_f$ with $T=T_\infty\left(1+\left(\theta_w-1\right)\theta\right)$ and R_d=0 Because there is no thermal radiation, this is the case. The final statement could potentially being made simpler to $\frac{\alpha(T_f-T_\infty)}{\Pr}\left[\left(1+R_d\left(1+(\theta_w-1)\theta^3\right)\theta'\right)\right]'$, Pr denotes the Prandtl number which is given by $\Pr=v_f/\alpha$

The dimensionless quantities in eq. (7), but also the boundary conditions in eq. (6), are introduced into the equations. Equation (1) is instantly satisfied, however equations (2), (3), and (4) form these System of ODEs:

$$f'''+ff''-f'^2+A^2+M(A-f')=0 \quad (8)$$

$$\frac{1}{\Pr}\left[\left(1+R_d\left\{1+(\theta_w-1)\theta\right\}^3\right)\theta'\right]'+f\theta'+N_b\theta'\varphi'$$
$$+N_t\theta'^2+E_c f''^2$$
$$+MEc\left(A-f'\right)^2+\lambda\theta=0 \quad (9)$$

$$\varphi''+Lef\varphi'+\frac{N_t}{N_b}\theta''=0 \quad (10)$$

Furthermore, when A = 0, the exact solution of Eq. (8) may be determined by using $f=(1-e^{-\sqrt{1+M\eta}})/\sqrt{1+M}$. Where C_f is the fluid's specific heat, T is the temperature, C is the concentration of nanoparticles, and D_T and D_B stand for the thermophoretic diffusion coefficient and Brownian motion, respectively. Where prime represents differentiation with respect to the function η, $M=\frac{\sigma_e B_0^2}{a\rho_f}$ is the magnetic parameter, $A=\frac{b}{a}$ is the ratio of the rates of free stream velocity to the velocity of the stretching sheet, $\lambda = \dfrac{Q}{aT_w \rho_{nf}}$, is the heat generation/absorption parameter.

Eqs. (8)– (10) Together with this boundary conditions

$$f(0)=0,\ f'(\infty)=A,\ f'(0)=1,\ \theta'(0)=-Bi[1-\theta(0)],$$
$$\theta(\infty)=0,\ \varphi(0)=1,\ \varphi(\infty)=0 \tag{11}$$

Some parameters in Eqs. (8)– (10) are expressed thus:

$$Ec=\frac{U_w^2(x)}{C_n(T_w-T_\infty)},\ Nb=\frac{\tau D_B(C_w-C_\infty)}{v_f},\ Bi=\frac{h(v_f/a)^{1/2}}{k},\ Nt=\frac{\tau D_B(T_f-T_\infty)}{v_f T_\infty},$$
$$\varphi(\eta)=C-C_\infty/C_w-C_\infty, Le=\frac{v_f}{D_B}.$$

The Eckert number (Ec), Brownian motion (Nb), Biot number (Bi), thermophoresis parameter (Nt), Lewis number (Le). The Nusselt number (Nu) is also considered as significant quantity. X-coordinate, as previously established, does not fit into the temperature calculation. As a result, we endeavor for the closest attainable distinct similarity solutions. The following are the values for the wall heat flux and wall mass flux, indicated by q_w and q_m:

$$q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}+(q_r)_w$$
$$=-k(T_w-T_\infty)(a/v_f)^{1/2}[1+N\theta_c^3]\theta'(0),$$
$$q_m=-D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}$$
$$=-D_B(C_w-C_\infty)(a/v_f)^{1/2}\varphi'(0). \tag{12}$$

By introducing the Nusselt number $Nu_x=\dfrac{xq_w}{k(T_f-T_\infty)}$ but the local Sherwood number $Sh=xq_m/D_B(C_w-C_\infty)$ and the relation becomes

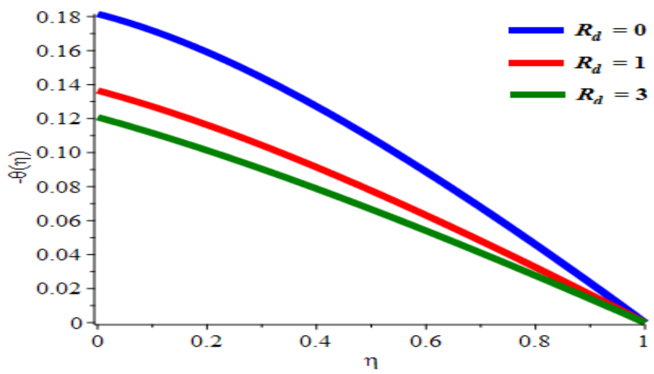
$$\frac{Nu_x}{\sqrt{\mathrm{Re}_x}}=-[1+R_d\theta_w^3]\theta'(0)=Nur,\ \frac{Sh}{\sqrt{\mathrm{Re}_x}}-\varphi'(0)=Shr \tag{13}$$

The local Reynolds number is $\mathrm{Re}_x=u_w(x)/v$.

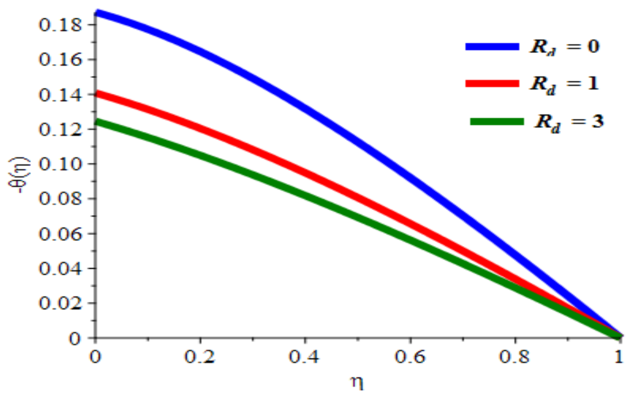
3. Results and Discussion

Here we will demonstrate how the result display the impact of important and significant parameters on momentum, temperature, nanoparticle concentration, and Nusselt number profiles. And used an expedient fourth order Runge–Kutta method as well as a shooting technique, the above flow model for the mentioned combined non-linear ODEs was investigated. For discrete values of the control parameters, such as the Lewis number Le, the radiation parameter R_d, the Prandtl number Pr, the Brownian motion parameter Nb, the thermophoresis parameter Nt and the magnetic parameter M, on Eqs. (8) – (10). Figures 2–10 show the data collected for the concentration, temperature, velocity, and Nusselt number curves. Whenever the Heat generation/absorption components were negated.

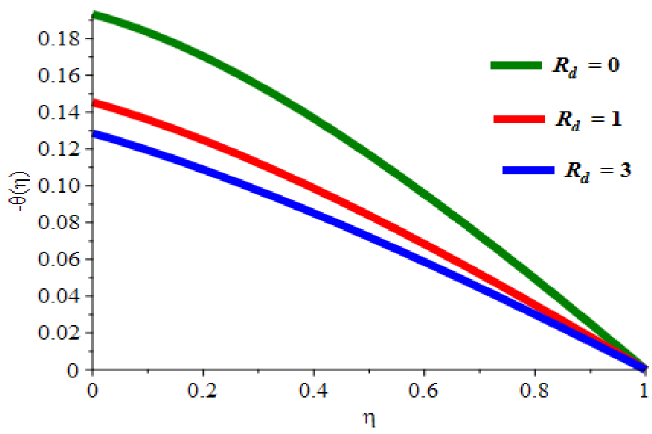
The effects of radiation parameter R_d are depicted on figure2 [(1a): heat absorption $\lambda<0$, (1b): no heat effect $\lambda=0$, (1c): heat generation $\lambda>0$] on the Nusselt number profile. In each of the three cases, it can be observed that for any increment in the radiation parameter it results in the increase in the temperature profile. Figure3 is showing the effects of the proportion on the momentum profile between the stretched sheet's velocity and the amount of free stream velocity [(2a): heat absorption $\lambda<0$, (2b): heat generation $\lambda>0$]. In the case where an increase in the parameter brings about an increase in the velocity of the fluid. Analysis on the consequences of the free stream velocity ratio to the velocity of the stretching sheet on the temperature profile [(3a): heat absorption $\lambda<0$, (3b): heat generation $\lambda>0$] is displayed on figure4. In the event when both the heat absorption parameter ($\lambda < 0$) and heat generation ($\lambda > 0$) are present increment of the values of A decreases the temperature field drastically.



1 (a)



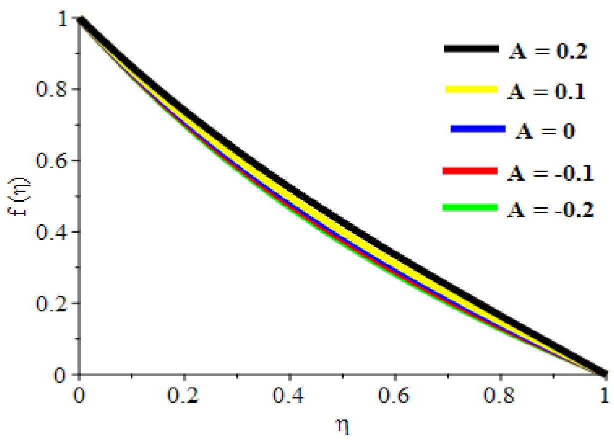
1 (b)



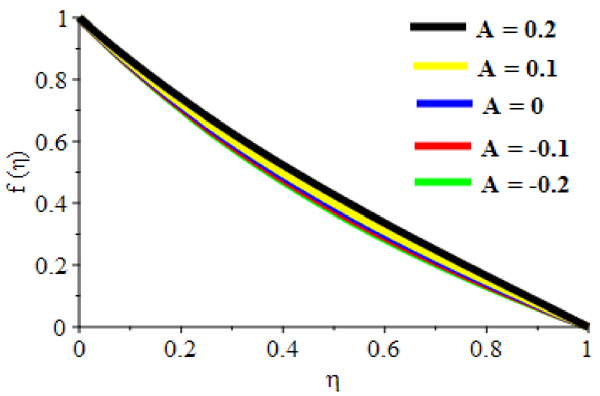
1 (c)

Figure 2. effects of radiation parameter on Nusselt number profiles for (1a) λ<0, (1b) λ=0, (1c) λ>0.

Figure5 discussed on the physical effects of the Biot number over the temperature profile in 3 faces of heat generation/ absorption as well as no heat effect. Such that, it is obvious that irrespective of heat generation/absorption or otherwise, Biot number exhibit the same property. Increase in the Biot number parameter brings about increment in temperature profile.

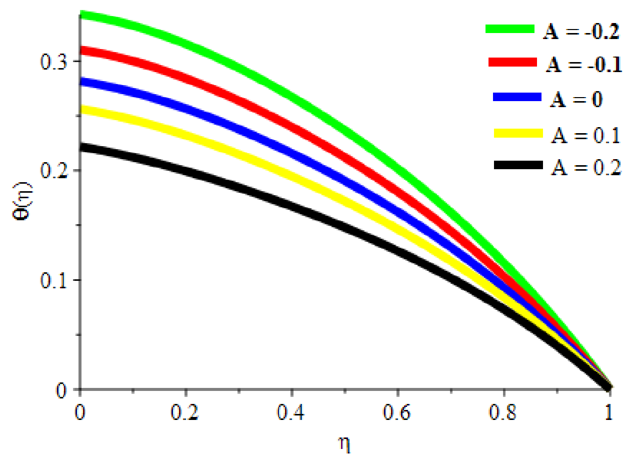


2 (a)

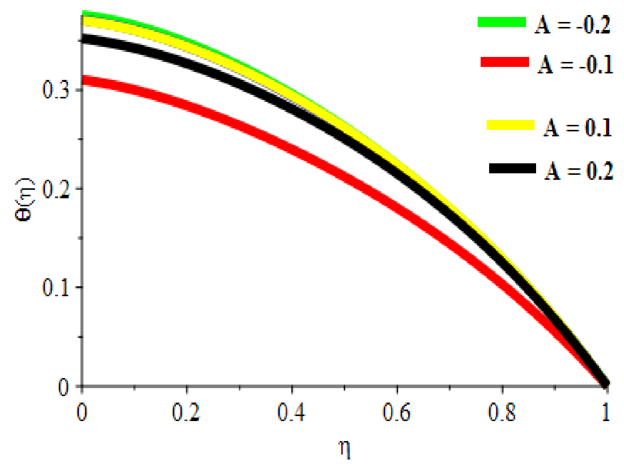


2 (b)

Figure 3. Effects of 'A' on momentum profile for (2a) λ<0, (2b) λ>0.

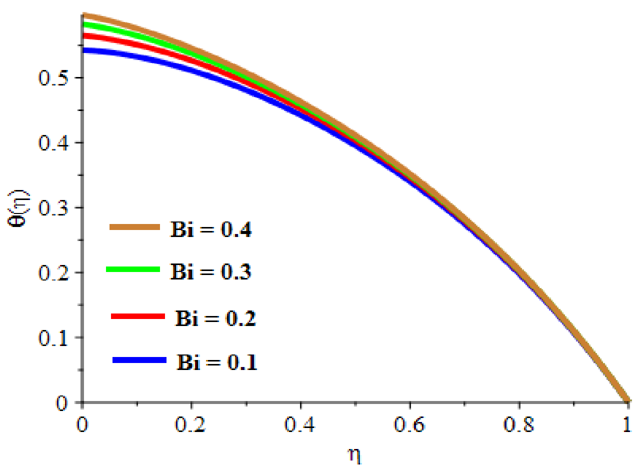


3 (a)

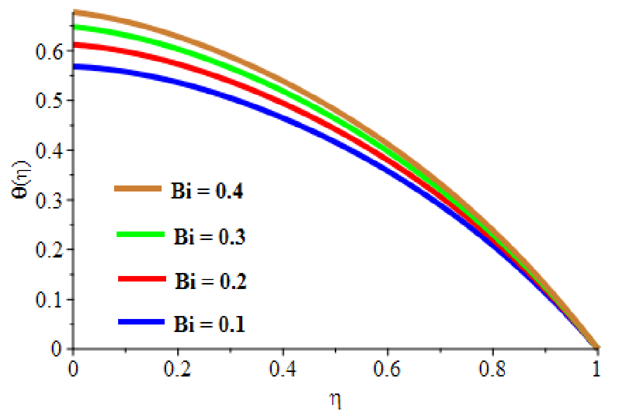


3 (b)

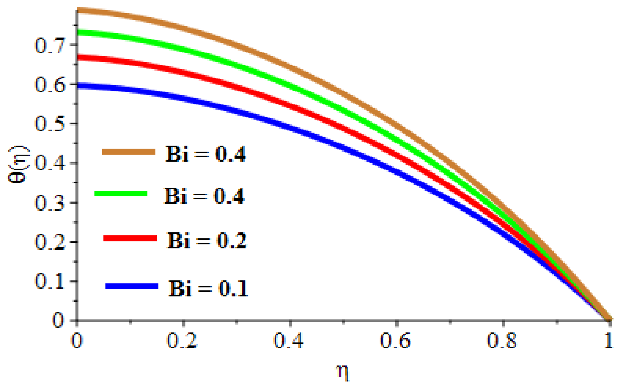
Figure 4. Effects of 'A' on temperature profile for (3a) λ<0, (3b) λ>0.



4 (a)



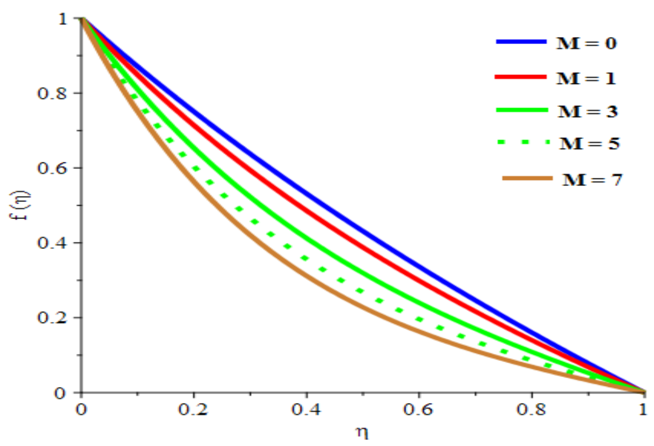
4 (b)



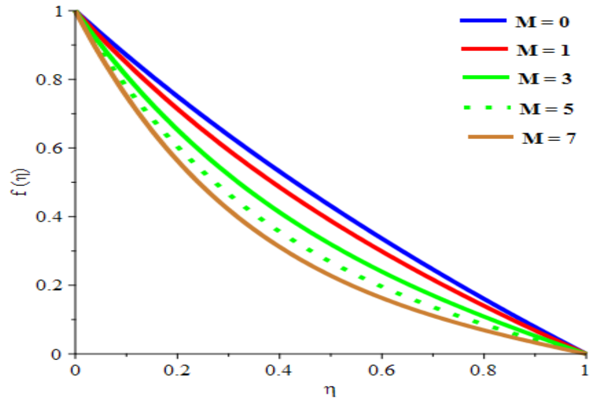
4 (c)

Figure 5. Effects of Bi on temperature profile for (3a) λ<0, (3b) λ=0, (3c) λ>0.

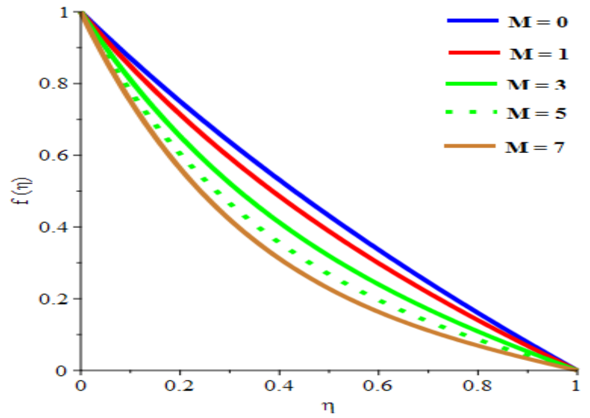
The physical effects of magnetic parameter over the momentum profile is displayed on figure6, where it is shown in 3 cases, heat absorption, no heat effect, as well as heat generation. In each case, increasing the magnetic (M) parameter slows down the fluid's momentum. Observing *figure* 7 closely will reveal the effects of magnetic parameter over the temperature profile, which is as well in 3faces, in case of heat absorption, no heat effect as well as heat generation. In both cases, increase in the magnetic parameter produces an increment in the temperature profile.



5 (a)

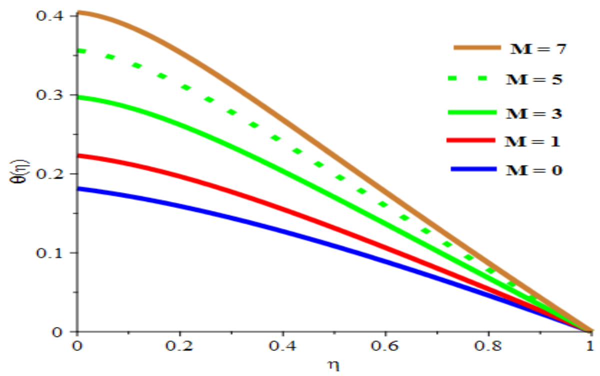


5 (b)

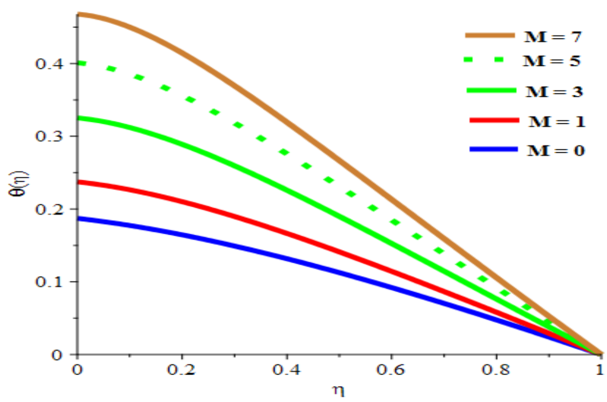


5 (c)

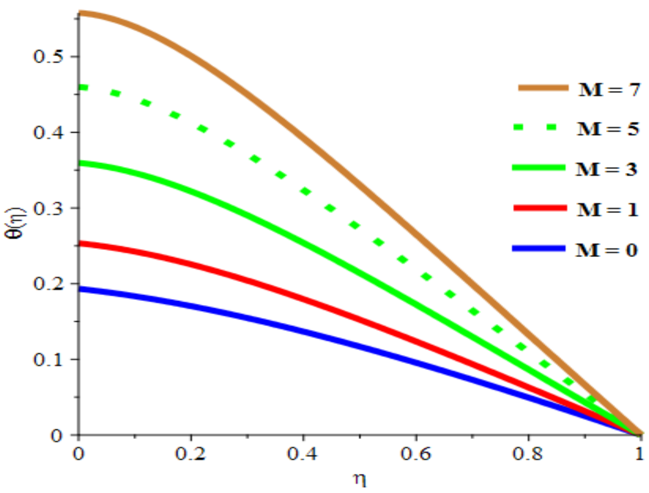
Figure 6. Effects of M on Momentum profile for (3a) λ<0, (3b) λ=0, (3c) λ>0.



6 (a)

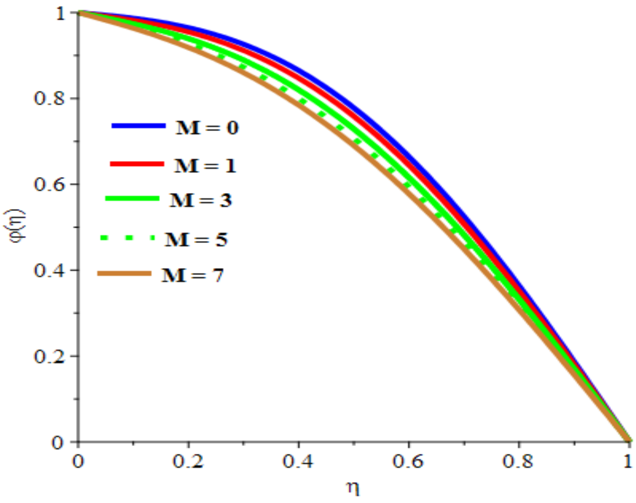


6 (b)

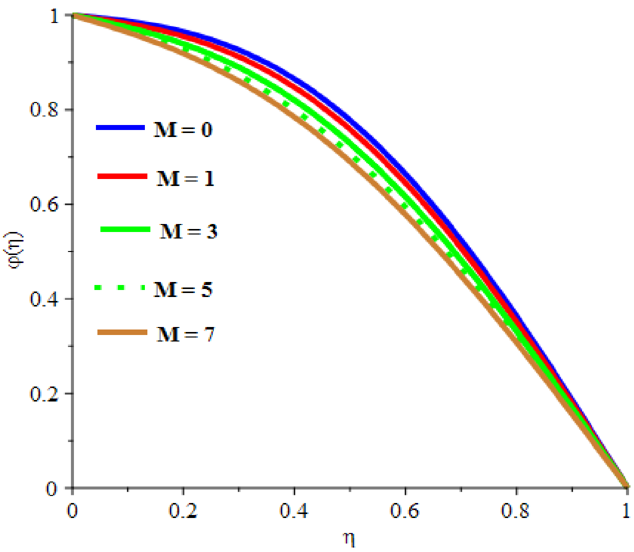


6 (c)

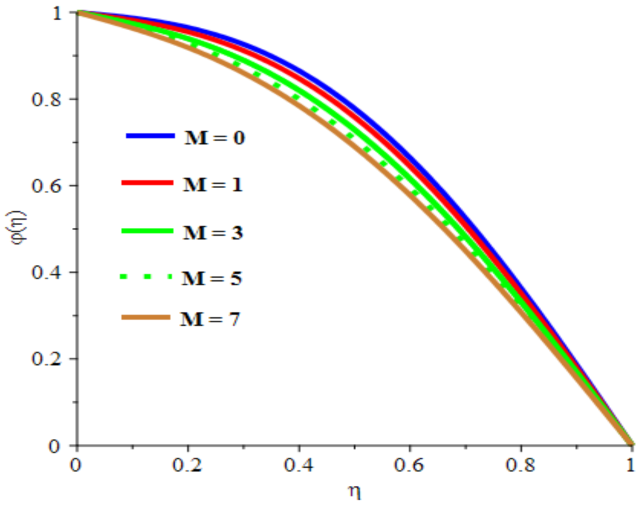
Figure 7. Effects of M on temperature profile for (3a) λ<0, (3b) λ=0, (3c) λ>0.



7 (a)



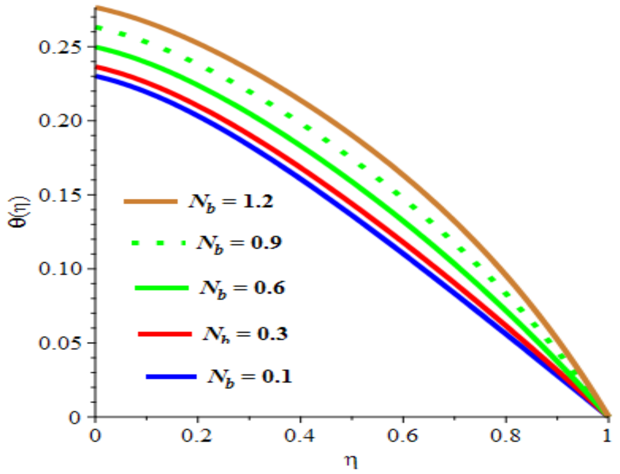
7 (b)



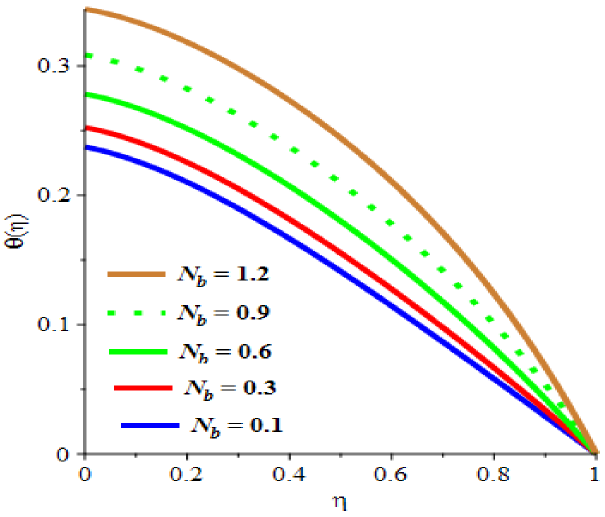
7 (c)

Figure 8. Effects of M on concentration profile for (3a) λ<0, (3b) λ=0, (3c) λ>0.

Figure8 depicted the effects of magnetic parameter on the nanoparticle concentration profile, which is explained in 3 faces: heat absorption, no heat effects, heat generation. In both three (3) cases, increment in the magnetic parameter reduces the nanoparticle concentration profile. Furthermore, the effects of Brownian motion on temperature profile is displayed on figure9. Which is discussed in 2 faces heat generation/absorption. In which it can be observed that increase in the Brownian motion parameter brings about increase in the temperature profile irrespective of either generation/absorption. Figure10 is based on the influence of Prandtl number on the temperature distribution profile, the analysis is concerned on 3 faces, heat absorption, no heat effect as well as heat generation. In both cases the temperatures is observed with the increase of the Prandtl number parameter.

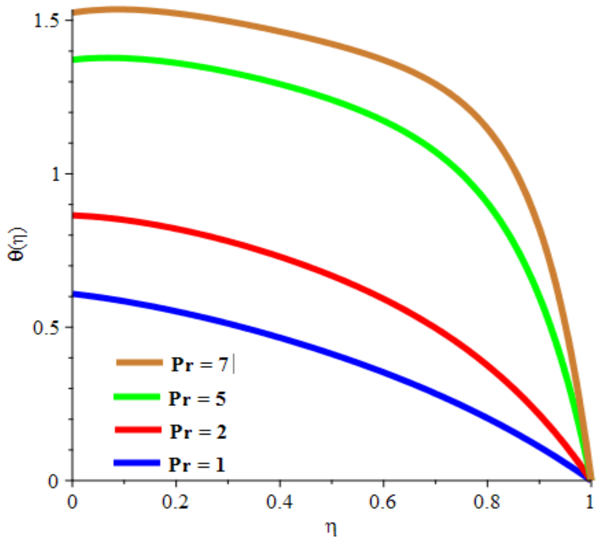


8 (a)

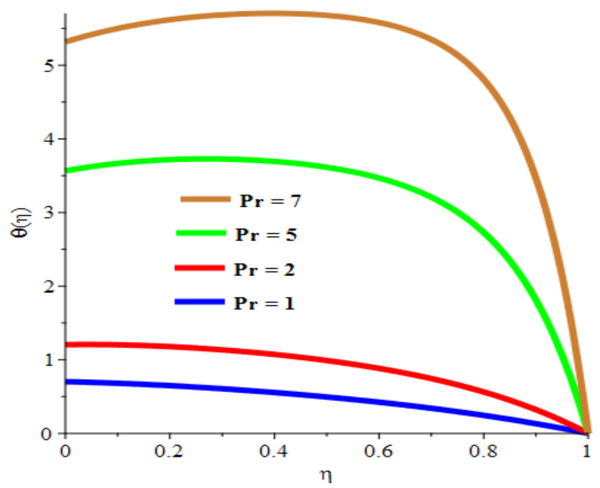


8 (b)

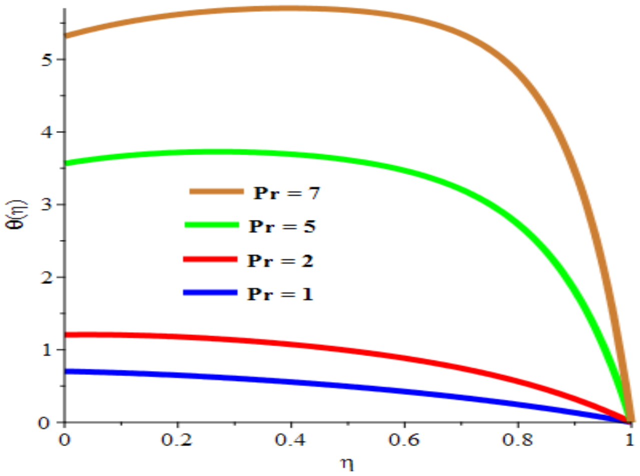
Figure 9. Effects of N_b on temperature profile for (8a) λ<0, (8b) λ>0.



9 (a)



9 (b)



9 (b)

Figure 10. Effects of Pr on temperature profile for (3a) λ<0, (3b) λ=0, (3c) λ>0.

$$g(\omega_i) = In\left(\frac{\omega_i}{1-\omega_i}\right) = \lambda(x_i) \quad (1)$$

$$\omega(x_i) = \frac{\exp\{\lambda(x_i)\}}{1+\exp\{\lambda(x_i)\}} \quad (2)$$

$$\lambda(x_i) = X_i^T \alpha = \begin{bmatrix} 1 & x_{1i} & x_{2i} & x_{1i}^2 & x_{2i}^2 \end{bmatrix} \quad (3)$$

6. Conclusion and Future Scope

In this article, we investigated heat transfer at the stagnation point of a nanofluid across a stretching surface in the presence of heat generation/absorption. The system of equations is modified into ordinary differential equations by employing appropriate similarity transformations. To solve the similarity equations numerically, the fourth order Runge–Kutta method together with shooting technique is being used. Contextual factors have a significant impact on momentum, temperature, concentration of nanoparticles, and Nusselt number profiles. In order to show how different physical traits affect temperature and velocity components, the gathered data is shown graphically. It was discovered that, Nusselt number, temperature and nanoparticle concentration are decreasing functions with respect to the heat generation/absorption parameter, whereas, momentum is found to be fluctuating. Furthermore, this research can be extended by considering the effects of impermeability on the momentum profile or the influence of chemical reaction on the concentration profile.

Data Availability
None.

Conflict of Interest
This is to declare that the Author (s) have no competing interests regarding the publication, research and authorship of this article.

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Authors' Contributions

Abubakar Assidiq Hussaini: Performed the experiments; Analyzed and interpreted the data; Wrote the first draft; as well as the camera ready paper.

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