

## Research Article

# Tsallis Holographic Dark Energy Cosmological Models in Modified Theory of Gravity

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**Abstract** — In the framework of  $f(R,T)$  gravity, this research aims to create Tsallis Holographic Dark Energy (THDE) models in a homogeneous and anisotropic Bianchi type-I model in Kasner form. We have employed two distinct volumetric expansions, Power law and exponential law, to determine the precise solutions of field equations. The conventional or conservative cosmological techniques, such as the deceleration parameter and the equation of state parameter, have been used to study the cosmic evolution. The physical characteristics of the model have been acquired. According to both expansion models, we have discovered that the universe is in the accelerating period and is reaching isotropy with delayed time. We also came to the conclusion that our model behaves similarly to a  $\Lambda$ CDM model.

**Keywords**— Modified gravity, Bianchi type-I space-time in Kasner form, THDE

## 1. Introduction

The remarkable progress in contemporary cosmology is shown in the current cosmic rapid expansion. Numerous cosmological observational studies have proposed that the highly accelerated scenario indicated by exotic force is the phenomenon underlying this amazing cosmic expanding paradigm or structure. The existence of some unobservable stuff or energy in this universe—something that is not just there but also controlling our universe—was the only plausible explanation that came to light. This element is referred to by scientists as "dark energy." With 68% of the current total energy in the observable cosmos, DE is the main component of the universe. It has been discovered that the holographic dark energy model (HDE) is a viable option for solving the puzzling DE riddle. In particular, the holographic principle [4–7], which is derived from black hole thermodynamics, has served as the foundation for numerous models of holographic dark energy [1–3]. This principle states that all of the physical quantities in the universe, including the density of dark energy, may be characterized by placing certain quantities on its edge. A novel class of dark energy models known as the Tsallis holographic dark energy model was created as a result of the Tsallis generalization provided by the Boltzmann-Gibbs entropy formula for black holes [8–9]. In this study, we have examined the Bianchi type I metric in Kasner form, motivated by the aforementioned talks. We assume that the cosmos is saturated with dark

matter and hypothetical isotropic fluids, such as the holographic dark energy components. We have examined the holographic dark energy of Tsallis.

The structure of this document is as follows: A research on Tsallis Holographic Dark Energy (THDE) models in an anisotropic and homogeneous Bianchi type-I model in Kasner form under the framework of  $f(R,T)$  gravity is introduced in Section 1. A review of certain scholars' work in the framework of  $f(R,T)$  gravity is presented in Section 2, which aided in the conduct of additional study for this publication. A Brief Methodology of  $f(R,T)$  Theory is presented in Section 3. The metric and field equation used to solve the problem are covered in Section 4. The Field Equation's exact solutions in two distinct scenarios and the development of associated cosmological models are presented in Section 5. Section 7 wraps up the study conducted for this paper and outlines future directions, while Section 6 presents the findings and debates.

## 2. Related Work

The new dark energy model, known as the Tsallis holographic dark energy (THDE) model [10–13], is based on the Tsallis generalization model provided by the Boltzmann-Gibbs entropy equation for black holes [8–9]. Karami and Khaledian [14] investigate a shift from the epitome to the illusory phase for entropy adjusted reconstructed models in

the context of  $f(R)$  gravity. To investigate the current cosmic growth, Houndjo and Piattella [15] replicated the HDE model in the  $f(R,T)$  model. In the framework of teleparallel theory, Daouda et al. [16] observed the HDE model for combining the significances of dark energy and dark matter. Jawad et al. [17] examined the HDE  $f(G)$  model's stability. In order to determine the quintessence and phantom span of the resulting model, Sharif and Zubair [18] recreated the HDE model and the agegraphic dark energy model in  $f(R,T)$ . Using Bianchi type-I, Fayaz et al. [19] examined the phantom and transient phases of cosmic history for reconstructed  $f(R,T)$  gravity models in the context of both new agegraphic dark energy and holographic dark energy.

### 3. A Brief methodology of $f(R, T)$ theory

The  $f(R,T)$  theory of gravity is a variation of General Relativity. The  $f(R,T)$  gravity field equations derived from the action,

$$s = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x,$$

where,  $f(R, T)$  is the arbitrary function of Ricci scalar  $R$ ,  $T$  is the trace of stress energy tensor of the matter  $T_{ij}$  and  $L_m$  is the matter Lagrangian density. Now the stress energy matter is defined as

$$T_{ij} = \frac{2}{\sqrt{-g}} \frac{\partial(-gL_m)}{\partial g^{ij}}$$

its trace is given by  $T = g^{ij}T_{ij}$  respectively. By assuming that  $L_m$  of matter depends only upon the metric components  $g_{ij}$  and not on its derivative, we obtain

$$T_{ij} = g_{ij}L_m - \frac{2\partial L_m}{\partial g^{ij}}$$

When the matter source is assumed to be a perfect fluid, the field equations of the  $f(R,T)$  theory of gravity provided by Harko et al. [20] for the function  $f(R,T)$  are

$$f(R, T)R_{ij} - \frac{1}{2}f(R, T)g_{ij} + g_{ij}(\Delta_i \Delta_j - \Delta_i \Delta_j)f(R, T) = 8\pi T_{ij} - f_T(R, T)T_{ij} - f_T(R, T)\theta_{ij}$$

where,.

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{kp} \frac{\partial^2 L_m}{\partial g^{ij} \partial g^{kp}},$$

$$f_R = \frac{\partial f(R, T)}{\partial R}, f_T = \frac{\partial f(R, T)}{\partial T}$$

The HDE model is the direct reduction of the Tsallis holographic dark energy model for  $\beta=1$ . We may determine the energy density of THDE using the Hubble horizon as the system's cutoff, which is  $\rho_T = BH^{4-2\beta}$ . Numerous scholars have lately examined THDE from a variety of angles; in particular, Tavayef et al. [21] examined the flat Friedmann-Robertson-Walker universe model within the context of THDE. Zadeh et al. [22] have studied a sign flexible

interaction between THDE and DE. In  $f(G,T)$ gravity, Sharif and Saba [23] developed the THDE model.

### 4. Metric and the field Equation

The Kasner space-time is an exact solution of the  $R_{ij} = 0$ , and the solutions of field equation plays an important role in the deliberation of some cosmological concerns. Consider the anisotropic Bianchi type-I metric or space time in Kasner form

$$ds^2 = dt^2 - t^{2p_1} dx^2 - t^{2p_2} dy^2 - t^{2p_3} dz^2, \tag{2}$$

Where,  $p_1, p_2$  and  $p_3$  are the three parameters that are requires to be constant, satisfying the relations  $p_1 + p_2 + p_3 = s$  and  $p_1^2 + p_2^2 + p_3^2 = \theta$ .

Assumed in this paper are hypothetical isotropic fluids like the holographic dark energy components and that the cosmos is saturated with matter.

For the matter source with Tsallis holographic dark energy, the energy momentum tensor is defined as

$$T'_{ij} = \rho_m u_i u_j; \overline{T}_{ij} = (\rho_T + P_T)u_i u_j + g_{ij}P_T \tag{3}$$

where  $\rho_T$  describes the Tsallis holographic dark energy,  $P_T$  indicates the pressure of the holographic dark energy, and  $\rho_m$  indicates the energy density of matter.

#### The Field Equation:

The field equation for  $f(R, T)$  gravity theory using equation (1) is given by,

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \tag{4}$$

where  $p$  is the matter source pressure for a perfect fluid and  $T$  is an arbitrary function that represents the trace of the stress energy tensor of matter. We can consult [20] for a thorough and methodical derivation of the  $f(R,T)$  gravity field equations. Let's think about

$$f(T) = \mu T \text{ and } T_{ij} = T'_{ij} + \overline{T}_{ij}.$$

Equations (2), (3), and (4) are used to obtain the extremely non-linear differential equation, which looks like this:

$$\left[ p_1(s-1) - \frac{1}{2}[s^2 - 2s + \theta] \right] t^{-2} = -(8\pi + 2\mu)M + [2p + T]\mu \tag{5}$$

$$\left[ p_2(s-1) - \frac{1}{2}[s^2 - 2s + \theta] \right] t^{-2} = -(8\pi + 2\mu)M + [2p + T]\mu \tag{6}$$

$$\left[ p_3(s-1) - \frac{1}{2}[s^2 - 2s + \theta] \right] t^{-2} = -(8\pi + 2\mu)M + [2p + T]\mu \tag{7}$$

$$\frac{1}{2}t^{-2}[s^2 - \theta] = (8\pi + 2\mu)M + [2p + T]\mu \tag{8}$$

Where,  $M = \rho_m + \rho_T + 2P_T$ .

### 5. Solutions of the Field Equation

The model's spatial Volume  $V$  is provided by

$$V = t^s \tag{9}$$

The generalized mean Hubble Parameter  $H$  is provided by, while the mean scale factor of the model is derived as

$$a = V^{\frac{1}{3}} = t^{\frac{s}{3}} \tag{10}$$

$$H = \frac{1}{3} \sum_{i=1}^3 H_i \tag{11}$$

The expansion scalar is given by  $\theta = 3H$  (12)

Using equations (5) and (6), we get

$$\frac{d}{dt} \left( \frac{p_1}{t} - \frac{p_2}{t} \right) + \left( \frac{p_1}{t} - \frac{p_2}{t} \right) \frac{s}{t} = 0 \tag{13}$$

Using equations (9) and (13), we yield

$$\frac{d}{dt} \left( \frac{p_1}{t} - \frac{p_2}{t} \right) + \left( \frac{p_1}{t} - \frac{p_2}{t} \right) \frac{v}{v} = 0 \tag{14}$$

On integrating, we get

$$\frac{p_1}{p_2} = D \exp \left( X \int \frac{1}{v} \right) \tag{15}$$

where,  $D$  and  $X$  are constant of integration.

Explicitly, one can write the metric potentials as follows

$$t^{p_1} = D_1 v^{\frac{1}{3}} \exp \left( X_1 \int \frac{1}{v} \right) \tag{16}$$

$$t^{p_2} = D_2 v^{\frac{1}{3}} \exp \left( X_2 \int \frac{1}{v} \right) \tag{17}$$

$$t^{p_3} = D_3 v^{\frac{1}{3}} \exp \left( X_3 \int \frac{1}{v} \right) \tag{18}$$

Where,  $D_i (i = 1,2,3)$  and  $X_i (i = 1,2,3)$  satisfy the relation  $D_1 D_2 D_3 = 1$  and  $X_1 + X_2 + X_3 = 0$ .

According to [21], the holographic dark energy's inequality form on the previously indicated energy density is  $L^3 \rho_d \leq L m_p^2$ . This format can be expressed as

$$\rho_d = \frac{3C^3 m_p^2}{L^2}$$

where  $L$  stands for the IR cutoff,  $C$  for the dimensionless magnitude, and  $m_p^2 = (8\pi G)^{-1}$  for the decreased Plank mass.

The horizon entropy of the black hole was modified by the authors in Ref. [22] to be  $S_\delta = \psi A_\delta$ , where  $\psi$  is an unknown value and  $\delta$  is the constant. In this case, the horizon's area is  $A = 4\pi L^2$ . The relationship between the IR ( $L$ ) cutoff, system entropy ( $S$ ), and UV ( $L$ ) cutoff was described by the authors in Ref. [23] as  $L^3 \Lambda^3 \leq (S)^{\frac{3}{4}}$ , which provides  $\Lambda^4 \leq (\psi (4\pi)^\delta) L^{2\delta-4}$ . Here, vacuum energy density is indicated by  $\Lambda^4$ . By using this inequality, the authors in [23] explained that the energy density of Tsallis holographic dark energy (THDE) is provided by since the Tsallis generalized entropy area relation is independent of the gravitational theory employed to examine the system.

$$\rho_T = B H^{4-2\delta} \quad \text{where, } B, \delta \text{ are constants} \tag{19}$$

We require an additional condition to fully solve the system because the field equations in (5)–(8) are extremely non-linear. Consequently, two distinct volumetric expansion rules can be written as

$$V = C t^{3n} \quad \text{where, } C, n \text{ are constants} \tag{20}$$

$$V = C e^{3p't} \quad \text{where, } C, p' \text{ are constants} \tag{21}$$

#### 5.1 Power law expansion model

Using equations (16)-(18) and (20), we get

$$t^{p_1} = D_1 C^{\frac{1}{3}} v^{\frac{1}{3}} \exp \left( X_1 \int \frac{1}{Ct^{3n}} dt \right) \tag{22}$$

$$t^{p_2} = D_2 C^{\frac{1}{3}} v^{\frac{1}{3}} \exp \left( X_2 \int \frac{1}{Ct^{3n}} dt \right) \tag{23}$$

$$t^{p_3} = D_3 C^{\frac{1}{3}} v^{\frac{1}{3}} \exp \left( X_3 \int \frac{1}{Ct^{3n}} dt \right) \tag{24}$$

The formula for the Mean Hubble parameter is

$$H = \frac{1}{Ct^{3n}} + \frac{1}{nt} \tag{25}$$

Thus, the holographic dark energy of Tsallis produces

$$\rho_T = B \left( \frac{1}{Ct^{3n}} + \frac{1}{nt} \right)^{4-2\delta} \tag{26}$$

The matter and THDE conservation equation is given by

$$\dot{\rho}_m + \dot{\rho}_T - 3H(\rho_T + P_T) = 0 \tag{27}$$

$$\dot{\rho}_T + 3H(\rho_T + P_T) = 0 \tag{28}$$

Equation (27) and (28) yields,

$$P_T = \frac{-B}{3} \left[ \frac{nt+Ct^{3n}}{nCt^{3n+1}} \right]^{2-2\delta} \left[ 4 - 2\delta - 3 \left( \frac{nt+Ct^{3n}}{nCt^{3n+1}} \right)^2 \right] \tag{29}$$

The expression for the THDE EoS parameter is

$$\omega_T = P_T \rho_T^{-1} \tag{30}$$

$$\therefore \omega_T = \frac{-1}{3} \left[ \frac{nt+Ct^{3n}}{nCt^{3n+1}} \right]^{-2} \left[ 4 - 2\delta - 3 \left( \frac{nt+Ct^{3n}}{nCt^{3n+1}} \right)^2 \right]$$

The deceleration parameter sheds light on the model's expansion. Cosmological models are divided into the following categories based on the Hubble parameter and the deceleration parameter in the presence of time:

The model is expanding and decelerating when  $H>0, q>0$ , expanding and accelerating when  $H>0, q<0$ , contracting and decelerating when  $H<0, q>0$ , contracting and accelerating when  $H<0, q<0$ , and expanding and accelerating when  $H>0, q=0$ . indicates that the model is expanding, with zero slowdown or continuous expansion,  $H<0$ , and  $q=0$ . indicates that the model is contracting with zero deceleration,  $H=0$ , and  $q=0$ . demonstrates that the model is static.

$$q = -1 + \frac{3n^2+Ct^{3n-1}}{nt+Ct^{3n}} \tag{31}$$

The energy density of matter is given by

$$\rho_m = \exp \left( \frac{-3t^{-3n+1}}{C(-3n+1)} \right) - \frac{t^3}{n} + C_1,$$

Where,  $C_1$  is the constant of integration.

The mean anisotropy Parameter  $\Delta$  is given by,

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \frac{(H_i - H)^2}{H} = 0 \tag{32}$$

Also the shear Scalar  $\sigma$  is defined as,

$$\sigma^2 = \frac{3}{2} \Delta H^2 = 0 \tag{33}$$

Equations (32) and (33) show that this model is both isotropic and shear free.

**5.2. Exponential law expansion model:**

Using equations (16)-(18) and (21), we get

$$t^{p_1} = D_1(Ce^{3p't})^{\frac{1}{3}} \exp\left(X_1 \int \frac{1}{Ce^{3p't}} dt\right) \tag{34}$$

$$t^{p_2} = D_2(Ce^{3p't})^{\frac{1}{3}} \exp\left(X_2 \int \frac{1}{Ce^{3p't}} dt\right) \tag{35}$$

$$t^{p_3} = D_3(Ce^{3p't})^{\frac{1}{3}} \exp\left(X_3 \int \frac{1}{Ce^{3p't} t^{3n}} dt\right) \tag{36}$$

One can determine the mean Hubble parameter as

$$H = \frac{1}{Ce^{3p't}} + \frac{1}{3} \tag{37}$$

The THDE's energy density turns into

$$\rho_T = B \left(\frac{1}{Ce^{3p't}} + \frac{1}{3}\right)^{4-2\delta} \tag{38}$$

The expression for the pressure of THDE is

$$P_T = \frac{-B}{3} \left[\frac{3+Ce^{3p't}}{3Ce^{3p't}}\right]^{2-2\delta} \left[4 - 2\delta - 3 \left(\frac{3+Ce^{3p't}}{3Ce^{3p't}}\right)^2\right]$$

The THDE EoS parameter is found out to be

$$\omega_T = P_T \rho_T^{-1} = \frac{-1}{3} \left[\frac{3+Ce^{3p't}}{3Ce^{3p't}}\right]^{-2} \left[4 - 2\delta - 3 \left(\frac{3+Ce^{3p't}}{3Ce^{3p't}}\right)^2\right]$$

The deceleration parameter is  $q = -1 + \frac{gp}{3+Ce^{3p't}}$

The energy density of matter is found to be

$$\rho_m = \exp\left(\frac{e^{-3p't}}{ep} - t\right) + C_1,$$

Where,  $C_1$  is constant of integration

The mean anisotropy Parameter  $\Delta$  is given by,

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \frac{(H_i - H)^2}{H} = 0 \tag{39}$$

The shear Scalar  $\sigma$  is expressed as,

$$\sigma^2 = \frac{3}{2} \Delta H^2 = 0 \tag{40}$$

This model was also found to be isotropic and shear-free using equations (39) and (40).

**6. Results and Discussion**

The following is a graphical depiction of the kinematical and physical qualities.

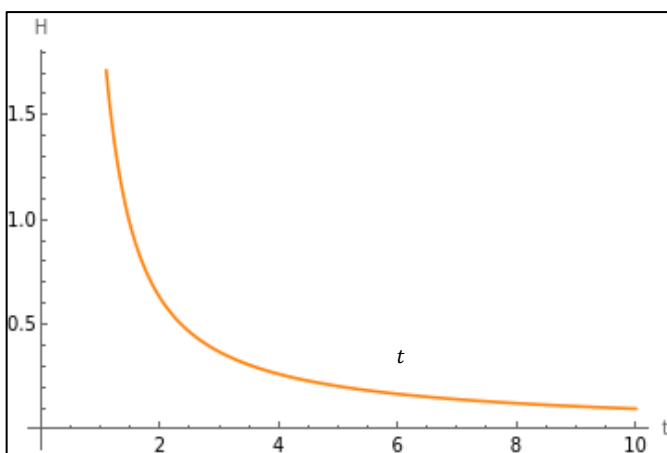


Figure 1: The Hubble parameter's change with regard to cosmic time is seen in Figure 1. for the power law model. When  $t \rightarrow \infty$ , we can say that  $H \rightarrow 0$ .

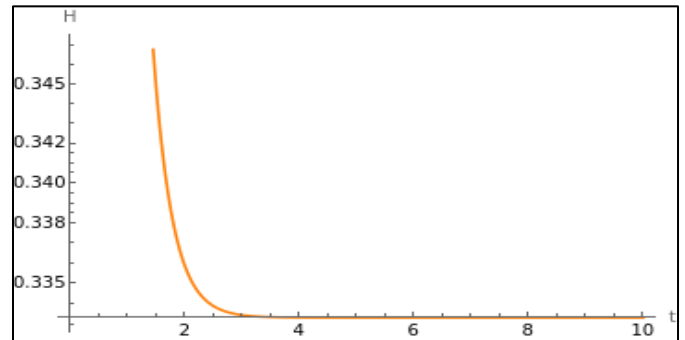


Figure 2: The Hubble parameter's fluctuation with cosmic time is shown in Figure 2. We can state that  $H \rightarrow 0$  for the exponential model when  $t \rightarrow \infty$ .

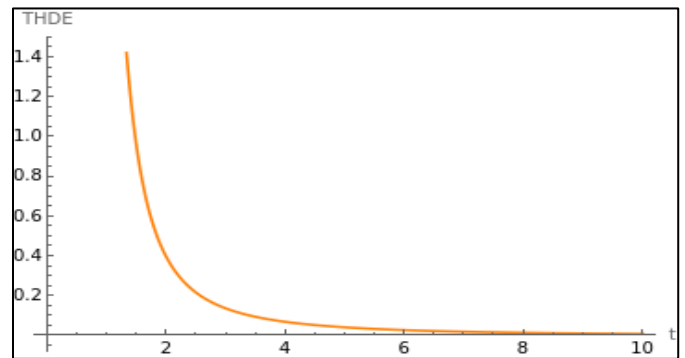


Figure 3: The energy density change of THDE with respect to cosmic time is shown in Figure 3. We can state that  $\rho_T \rightarrow 0$  for the power law model for  $t \rightarrow \infty$ . In this case, the THDE  $\rho_T$  energy density rapidly drops.

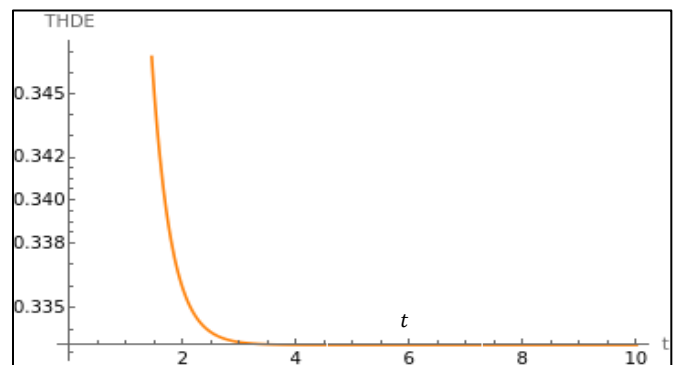


Figure 4: The energy density of THDE varies with time, as seen in Figure 4. In the volumetric exponential model, we can argue that  $\rho_T$  drops more quickly than power law when  $t \rightarrow \infty$ .

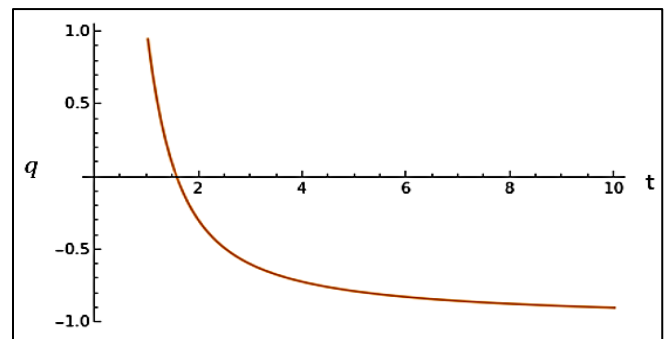
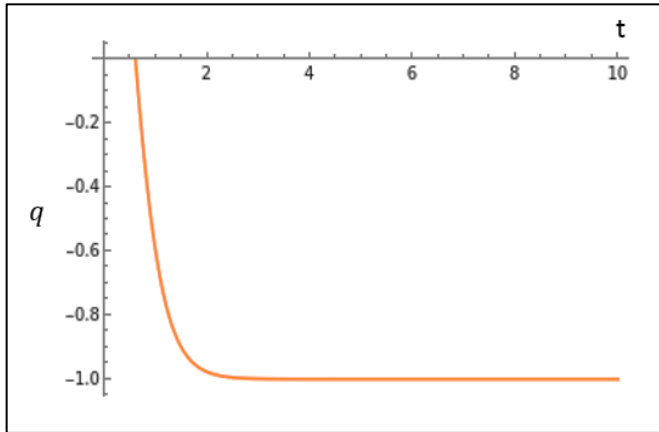
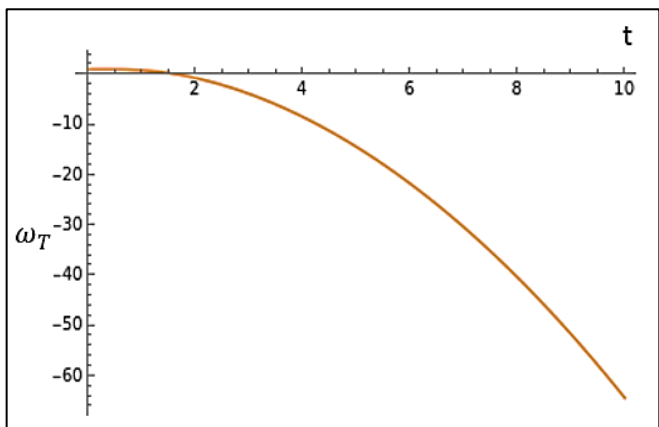


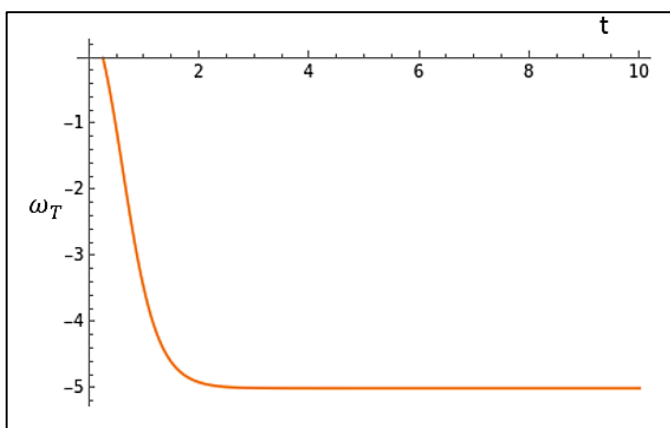
Figure 5: The deceleration parameter's fluctuation with cosmic time is depicted in Figure 5. The universe's acceleration or slowdown is explained by the deceleration parameter. The model inflates when the deceleration parameter has a negative value; a decelerating universe is indicated by a positive value. Additionally, we noticed that the universe changed its deceleration parameter, and that there was a shift from deceleration to acceleration.



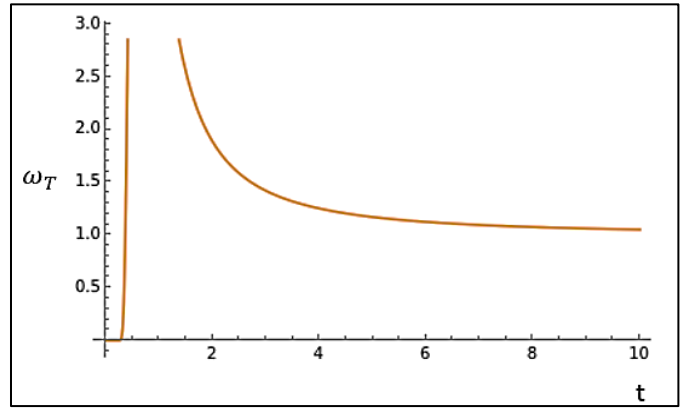
**Figure 6:** The deceleration parameter's fluctuation with cosmic time is depicted in Figure 6. The universe's current fast expansion is indicated by a negative sign of the deceleration parameter at delayed times, while a positive sign of the parameter denotes a deceleration. The developed model's deceleration parameter's negative value suggests that the cosmos is expanding and speeding up.



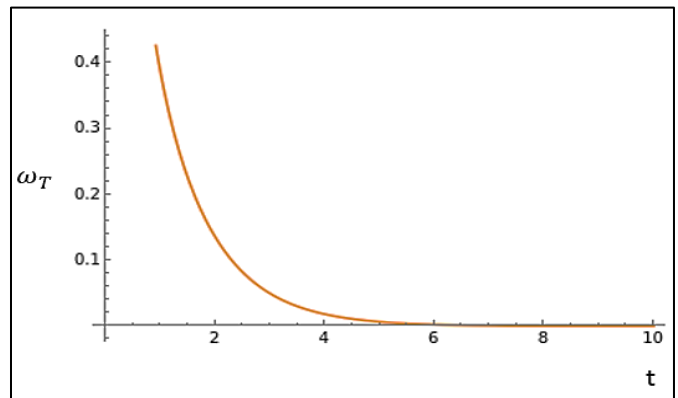
**Figure 7:** The THDE EoS parameter's change with cosmic time is depicted in Figure 7. According to several relativists, the EoS of dark energy transmission changed from  $w_T > -1$  in the quintessence area, or the near past, to  $w_T < -1$  in the phantom region, or the more recent stage. The EoS value fluctuates throughout the range, as Figure 7 makes evident, and this is consistent with the SNIa and CMB measurements.



**Figure 8:** The THDE EoS parameter's fluctuation with cosmic time is depicted in Figure 8. The resultant DE model is consistent with both current findings and the well-established theoretical result. From a higher negative number, it subtly increases with increasing time.



**Figure 9:** shows the variation of THDE EoS parameter against cosmic time.



**Figure 10:** shows the change of THDE EoS parameter with respect to cosmic time.

### 7. Conclusion and Future Scope

In this article, we have considered Bianchi type-I metric or space time in the Kasner form in the presence of THDE with the framework of  $f(R, T)$  i.e. modified theory of gravity. Exponential and power law volumetric expansion has been assumed to find the solutions of the field equations. We notice that,  $H$  (Hubble Parameter) is seems infinite at an initial stage in both the models and steadily declines to zero as the time increases. It means that, the universe expansion rate is very lower.

We observed that if  $\delta = 2$ ,  $\rho_T$  is constant in both the expansions. The anisotropy parameter  $\Delta$  is the function of  $t$  i.e. time in both the models, and it is 0. For both models power law and exponential model, the THDE density i.e.  $\rho_T$  is constant for  $\delta = 2$  and matter's density tends to be 0(zero) at late times. The deceleration parameter is seems to be negative for both the models, so that the universe is in the expanding phase at the late times. It is interesting to know that our model is isotropic and Shear free.

This work can be extended in other models of modified theories of gravitation and we hope that we will present a better results to understand the nature of universe and universe's growth.

#### Conflict of Interest

The Authors reports no any conflict of interest.

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Funding agencies have not provided any assistance for this research project.

### Authors' Contributions

Author 1 developed the study and completed the literature review. Author 2 created the manuscript's first draft, performed computations, illustrated every graph, and conducted data analysis. The third author, who worked on the protocol's creation, illustrated the findings. Following their contributions to the manuscript's review, rewriting, and editing, all authors endorsed the final draft.

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