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Research Paper

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f(R) Induced Gravity from the Occurrence of a Binary Action

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***Abstract*—** In this paper, we generalize the recently discovered relationship between induced gravity and a binary action to the extent of constructing the cosmological model for the late time cosmic acceleration from the corresponding f(R) induced gravity model. We have first constructed the action and then derive the field equations by conventional approach. The derived field equations are then applied to FLRW metric and modified Friedman's equations are derived.

***Keywords*—** *f(R) gravity, binary action, FLRW metric.*

I. INTRODUCTION

General Relativity is one of the most successful theories of gravity when it comes to describe the large-scale structure of our universe. The underlying mathematics is without doubt much more complicated when it comes to solving equations but they provide exact and promising results. General Relativity also have wide applications in Cosmology and Astrophysics such as it's observation of the expanding universe and the presence of dark matter and dark energy.

The discovery of expanding universe and the existence of dark matter and dark energy has triggered a tremendous shift in our attention and understanding of how universe works. But Einstein's field equations fail when they are applied to the early stages of our universe and dark matter and dark energy. It was obvious back then that a new theory of gravity was needed to describe the current structure of our universe. Many researchers since then have tried to resolve this problem and alternative model of gravity are discovered which are more generalized than Einstein's theory of gravity [1]-[4].

One among these modified theories is the f(R) theory of gravity where the Ricci scalar R in Einstein-Hilbert action is replaced by an arbitrary function f(R) which is a function of the Ricci scalar. This has been so far one of the most successful theories of modified gravity and have numerous applications in Cosmology.

One another theory of gravity that has been used to study Cosmology is the induced theory of gravity. An induced theory of gravity using a binary action was first constructed by El-Nabulsi in 2017 [5]. This theory was then subsequently applied Cosmology to derive the corresponding modified Friedman's equations. We will refer to this theory of gravity as binary induced theory of gravity or simply binary induced gravity. More literature on induced gravity can be found in the references [7]-[11].

The main aim of this paper is to construct a binary induced f(R) gravity where we try to unify the notion of f(R) gravity with the binary induced gravity in a single framework. The paper is organized as follows. In section 2, we write the action and then derive the corresponding field equations. A scalar tensor representation of our derived field equation is also presented. In section 3, we solve our derived field equations and derive the modified Friedman's equations.

II. ACTION AND FIELD EQUATIONS

Consider the action functional of the form [5]

$$S = \int \sqrt{-g}L_m d^4x + \log\left\{e^{a\int\frac{1}{2\kappa}\sqrt{-g}Rd^4x-\lambda(x)\left(\sqrt{-g}R-\epsilon\right)} - b\right\} \tag{2.1}$$

where a, b are any real constants, L_m is the Lagrangian for matter field, $k = 8\pi G$ and $\lambda(x)$ is the Lagrange's multiplier introduced by forcing the constraint $(-g)^{1/2}$ R = ϵ. Where ϵ is any constant real or imaginary. This constraint was introduced so that it results in an action which is invariant under the full diffeomorphism. Variation of the action 2.1 yields the following induced gravity field equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{2a\kappa' e^{a\epsilon V_4}}{e^{a\epsilon V_4} - b} T_{\mu\nu} \tag{2.2}$$

where κ' is such that 1/ κ' = 1/ κ-2λ and V_4 is the proper volume defined by

$$V_4 = \int \sqrt{-g} d^4x \tag{2.3}$$

which is constant. In the presence of the cosmological constant Eqn. (2.2) becomes

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{2a\kappa'^{e^{a\epsilon V_4}}}{e^{a\epsilon V_4} - b} T_{\mu\nu}. \tag{2.4}$$

We now introduce the corresponding *f*(R) gravity action that would give us the field equations for the binary induced *f*(R) gravity. Since in *f*(R) theory of gravity, we replace Ricci scalar R it an arbitrary function *f*(R) of Ricci scalar, then consider action functional of the form

$$S = \int \sqrt{-g} L_m d^4x + \log\left\{e^{a\int \frac{1}{2\kappa}\sqrt{-g} f(R) d^4x - \lambda(x)\left(\sqrt{-g} f(R) - \epsilon\right)} - b\right\} \tag{2.5}$$

where λ(x) is the Lagrange's multiplier introduced by forcing the constraint $(-g)^{1/2}$ *f*(R) = ∈. In our formalism, we can write the variation of our action S with respect to the metric tensor as

$$\frac{dS}{dg^{\mu\nu}} = \frac{\delta S}{\delta g^{\mu\nu}} + \frac{\delta S}{\delta S_1}\frac{dS_1}{dg^{\mu\nu}} + \frac{\delta S}{\delta S_2}\frac{dS_2}{dg^{\mu\nu}} \tag{2.6}$$

where

$$S_1 = \int \sqrt{-g} L_m d^4x \tag{2.7}$$

$$S_2 = \int \frac{1}{2\kappa}\sqrt{-g} f(R) d^4x - \lambda(x)\left(\sqrt{-g} f(R) - \epsilon\right). \tag{2.8}$$

Since S is independent of the metric tensor, we can set the first term in Eqn. (2.6) equal to zero. This gives

$$\frac{dS}{dg^{\mu\nu}} = \frac{\delta S}{\delta S_1}\left(\int \frac{1}{2\kappa}\frac{\delta}{\delta g^{\mu\nu}}\sqrt{-g} f(R) d^4x - \frac{\delta}{\delta g^{\mu\nu}}\lambda(x)\left(\sqrt{-g} f(R) - \epsilon\right)\right) + \frac{\delta S}{\delta S_2}\left(\int \frac{\delta}{\delta g^{\mu\nu}}\sqrt{-g} L_m d^4x\right). \tag{2.9}$$

Solving for the variation yields

$$\frac{dS}{dg^{\mu\nu}} = \frac{1}{2}\sqrt{-g}\int\left(\frac{\delta S}{\delta S_1}\left(\frac{1}{\kappa} - 2\lambda\right)\left(f'(R)R_{\mu\nu} - \frac{f(R)}{2}g_{\mu\nu} - \left[\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla_\rho\nabla^\rho\right]f'(R)\right) + 2\left[\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla_\rho\nabla^\rho\right]\lambda f'(R) - \frac{\delta S}{\delta S_2}T_{\mu\nu}\right)\sqrt{-g} d^4x. \tag{2.10}$$

Applying ∇^μ to the above equation yields $R\nabla^\mu\left(\lambda f'(R)\right) = 0$. Therefore, as long as $R \neq 0$, $\lambda f'(R)$ is a constant. Thus, using the fact that

$$\frac{\delta S}{\delta S_1} = 1 \tag{2.11}$$

and

$$\frac{\delta S}{\delta S_2} = \frac{2a e^{a\int\sqrt{-g} f(R) d^4x}}{e^{a\int\sqrt{-g} f(R) d^4x} - b} = \frac{2a e^{a\epsilon V_4}}{e^{a\epsilon V_4} - b} \tag{2.12}$$

we get the following field equation:

$$f'(R)R_{\mu\nu} - \frac{f(R)}{2}g_{\mu\nu} - \left[\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla_\rho\nabla^\rho\right]f'(R) = \frac{2a\kappa' e^{a\epsilon V_4}}{e^{a\epsilon V_4} - b} T_{\mu\nu} \tag{2.13}$$

where V_4 is the proper volume defined by $dV_4 = \sqrt{-g} d^4x$. To write the equation in a simpler form, we introduce the following operator:

$$\hat{P}_{\mu\nu} = \nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla_\rho\nabla^\rho. \tag{2.14}$$

The field equation then takes the form

$$f'(R)R_{\mu\nu} - \frac{f(R)}{2}g_{\mu\nu} - \hat{P}_{\mu\nu}f'(R) = \frac{2a\kappa' e^{a\epsilon V_4}}{e^{a\epsilon V_4} - b}T_{\mu\nu}. \tag{2.15}$$

Adding and subtracting the term $f'(R)g_{\mu\nu}R/2$ from the above equation yields

$$G_{\mu\nu} + G_{\mu\nu}^{dark} = \frac{1}{f'(R)}\frac{2a\kappa' e^{a\epsilon V_4}}{e^{a\epsilon V_4} - b}T_{\mu\nu} \tag{2.16}$$

where $G_{\mu\nu}$ is the Einstein's tensor defined by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \tag{2.17}$$

and

$$G_{\mu\nu}^{dark} = \frac{1}{f'(R)}\left\{\frac{1}{2}[f'(R)R - fR)]g_{\mu\nu} - \hat{P}_{\mu\nu}f'(R)\right\}. \tag{2.18}$$

If we want to write field equations of *f*(R) induced gravity into a scalar-tensor representation, then we need to introduce the following Legendre transformation

$$\{R, f\} = \{\phi, V\} \tag{2.19}$$

defined as

$$\phi = f'(R),\ \ V(\phi) = R(\phi)f'(R) - f\big(R(\phi)\big). \tag{2.20}$$

Field equation 2.16 then can be rewritten as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{\phi}\frac{2a\kappa' e^{a\epsilon V_4}}{e^{a\epsilon V_4} - b}T_{\mu\nu} + \theta_{\mu\nu} \tag{2.21}$$

where

$$\theta_{\mu\nu} = -\frac{1}{2}V(\phi)g_{\mu\nu} + \frac{1}{\phi}\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla_\rho\nabla^\rho\right)\phi. \tag{2.22}$$

Using the new variables, the trace of the above equation can be written as

$$3\nabla_\rho\nabla^\rho\phi + 2V(\phi) - \varphi\frac{dV}{d\phi} = \frac{2a\kappa' e^{a\epsilon V_4}}{e^{a\epsilon V_4} - b}T. \tag{2.23}$$

We could write the right-hand side of the above equation in another form by taking into account the assumption that our universe is an expanding sphere. Since $\sqrt{-g} = a^2(t)$,

$$V_4 = \int\sqrt{-g}d^4x = \int_0^{L_\infty} 4\pi r^2 dr\int_0^t dt = 32\pi L_\infty^3 t/3 \tag{2.24}$$

where L_∞ represents the maximum scale in the universe taken as the universe scale $L_\infty = r^*$ where r^* represents the physical radius of the null hypersurface. Thus

$$\frac{2a\kappa' e^{a\epsilon V_4}}{e^{a\epsilon V_4} - b} = \frac{2a\kappa' e^{32a\epsilon\pi L_\infty^3 t/3}}{e^{32a\epsilon\pi L_\infty^3 t/3} - b}. \tag{2.25}$$

III. Late Time Cosmic Acceleration

Consider the FLRW metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \tag{3.1}$$

Eqn. (2.13) for the above metric gives

$$3\left(\frac{\dot a}{a}\right)^2 - \frac{1}{f'(R)}\left\{\frac{1}{2}[f(R) - Rf'(R)] - 3\left(\frac{\dot a}{a}\right)\dot R f''(R)\right\} = \frac{2a\kappa' e^{a\epsilon V_4}}{e^{a\epsilon V_4} - b}\rho \tag{3.2}$$

and

$$\left(\frac{\ddot a}{a}\right) + \frac{1}{2f'(R)}\left\{\frac{\dot a}{a}\dot R f''(R) + \ddot R f'(R) + \dot R^2 f'''(R) - \frac{1}{3}[f(R) - Rf'(R)]\right\} = -\frac{a\kappa' e^{a\epsilon V_4}}{3e^{a\epsilon V_4} - b}(\rho + 3p) \tag{3.3}$$

where we have taken into account the perfect fluid description for matter given by

$$T_\nu^\mu = \mathrm{diag}\big(-\rho(t)c^2, p(t), p(t), p(t)\big). \tag{3.4}$$

Here, ρ denotes the usual mass density of the cosmic fluid and p denotes thermodynamical pressure. Eqn. (3.2) and (3.3) can also be written into the form

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{2}{3}\frac{a\kappa' e^{a\epsilon V_4}}{e^{a\epsilon V_4}-b}\left(\rho+\rho_{(c)}\right) \tag{3.2}$$

and

$$\left(\frac{\ddot{a}}{a}\right)=-\frac{a\kappa' e^{a\epsilon V_4}}{3e^{a\epsilon V_4}-b}\left(\rho+\rho_{(c)}+3\left(p+p_{(c)}\right)\right) \tag{3.3}$$

where $\rho_{(c)}$ and $p_{(c)}$ are defined as

$$\rho_{(c)}=\frac{e^{a\epsilon V_4}-b}{2a\kappa' e^{a\epsilon V_4} f'(R)}\left\{\frac{1}{2}[f(R)-Rf'(R)]-3\left(\frac{\dot{a}}{a}\right)\dot{R}f''(R)\right\} \tag{3.4}$$

and

$$p_{(c)}=\frac{e^{a\epsilon V_4}-b}{2a\kappa' e^{a\epsilon V_4} f'(R)}\left\{\frac{\dot{a}}{a}\dot{R}f''(R)+\ddot{R}f'(R)+\dot{R}^2f'''(R)-\frac{1}{3}[f(R)-Rf'(R)]\right\}. \tag{3.5}$$

Consider Eqn. (3.3), in the absence of matter i.e. $\rho=p=0$, wean obtain the late time cosmic acceleration if $\rho_{(c)}+3p_{(c)}<0$ is for any value of *f*(R). Upon further examination, one can notice that the value of the parameters ω_{eff} and α are same as those in the standard *f*(R) gravity [6] i.e. For the particular choice of $f(R)\propto R^n$ and $a(t)=a_0(t/t_0)^\alpha$, we get

$$\omega_{eff}=\frac{-6n^2-7n-1}{6n^2-9n+3},\ \ n\neq 1 \tag{3.6}$$

and

$$\alpha=\frac{-2n^2+3n-1.}{n-2} \tag{3.7}$$

For the suitable choice of n for which $\omega_{eff}<-1/3$, we achieve the late time cosmic acceleration.

IV. Conclusion

In this paper, we have constructed a *f*(R) gravity model of the induced gravity corresponding to a binary action of the form $S=S_1+\log\{e^{as_2}-b\}$. The corresponding field equations are derived from a conventional variational approach and a scalar-tensor representation of our derived field equations is also presented. We then solve the field equations by applying it to FLRW metric and using this modified Friedman's equations are derived. We have concluded that the value of ω_{eff} and α coincides with the value of the standard *f*(R) cosmology. Thus, the present formalism can also be taken into account while studying cosmology and with the presence of extra new terms coupled to the energy momentum, it will give us a more precise understanding of the corresponding cosmological applications. These cosmological applications are under progress and will be presented in another paper.

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