International Journal of Scientific Research in Physics and Applied Sciences
Vol.11, Issue.4, pp.01-12, August 2023
E-ISSN: 2348-3423
Available online at: www.isroset.org

Research Paper

Anisotropic Bianchi Type-V Perfect Fluid Cosmological Models in *f(R, T)* Gravity

Kalpana Pawar[1], N.T. Katre[2*]

[1]Dept. of Mathematics, Shivaji Science College, Nagpur, Dist. Nagpur (M.S.), India
[2]Dept. of Mathematics, Nabira Mahavidyalaya, Katol, Dist. Nagpur (M.S.), India

**Corresponding Author:* narendra.katre@rediffmail.com

Received: 22/Jun/2023; **Accepted**: 20/Jul/2023; **Published**: 31/Aug/2023. | **DOI**: https://doi.org/10.26438/ijsrpas/v11i4.112

Abstract— This paper deals with the study of anisotropic Bianchi type-V cosmological model filled with perfect fluid in the framework of *f(R, T)* gravity, where *R* and *T* are the Ricci scalar and trace of the energy-momentum tensor respectively. To solve the field equations completely, we have considered two different cases: (i) the expansion scalar of the space-time is proportional to the shear scalar (Collins et al., 1980) which gives a relationship between metric potentials and (ii) the law of variation of Hubble's parameter proposed by Berman (1983) which yields the constant deceleration parameter. We have studied some physical and kinematical parameters of both the models and discussed their behavior graphically. The constructed cosmological models are singularity free, expanding and do not approach isotropy throughout the evolution of universe. In the first case, universe decelerates in a standard way, while the second model represents both decelerating and accelerating phase of expansion. Also, the energy conditions are discussed for both the models.

Keywords— Bianchi type-V, *f(R, T)* gravity, Perfect fluid, Anisotropic, Energy conditions.

1. Introduction

In recent times, the discovery of an accelerating expansion of the universe has attracted much attention of the researchers. The astrophysical observations such as high redshift supernova experiment [1-5], Wilkinson Microwave Anisotropy Probe (WMAP) experiment [6, 7], fluctuation of cosmic microwave background radiation (CMBR) [8, 9] and large scale structure (LSS) [10, 11], and Baryon Acoustic Oscillations (BAO) [12] indicate an accelerating expansion of the universe caused by the presence of some kind of repulsive force in the universe which is repelling the cosmic objects farther apart, and suggest that this cosmic acceleration is driven by mysterious form of energy with a large negative pressure, termed as dark energy (DE) [13-18].

Einstein's general theory of relativity (GTR) is an amazing achievement in the modern physics, which has pioneered the use of modern mathematics in physical theories. The GTR has established itself as a very successful theory in describing the gravitational phenomena, dynamics of the solar system, and evolution of the universe, through its gravitational field equations and construction of cosmological models. Amazingly, the large scale structure of the universe has been still best described by the field equations of GTR. However, it is said that this theory does not fully account for certain aspects of present day cosmology. For example, GTR does not fully incorporate Mach's principle, it does not avoid singularity problem and does not explain the modern scenario of accelerated expansion of the universe. Hence several theories of gravitation have been proposed as alternative to GTR. In order to study and explain the accelerating expansion of the universe, two methods have been proposed; one way is to investigate various dark energy candidates playing major role due to modification in the energy momentum tensor in the field equations of GTR and the other is to modify GTR. Einstein was the first who gave the concept of dark energy by introducing the cosmological constant Λ which is now considered as the most suitable candidate for dark energy. In recent years several probable candidates for dark energy, namely, quintessence [19-23], *k*-essence [24-26], phantom energy [27, 28], quintom [29, 30], tachyon [31, 32], chameleon [33], holographic dark energy (HDE) [34-37], Ricci DE [38], new age graphic DE [39, 40], Chaplygin gas [41], extended Chaplygin gas [42, 43] and the generalized Chaplygin gas [44], etc. characterized by their equation of state (EoS) parameter $\omega = p_\Lambda / \rho_\Lambda$ have been proposed and accordingly the cosmological models are being constructed and studied. In the second approach to explain the accelerating expansion of the universe, in the recent past, many modified theories of gravity have been developed by altering Einstein-Hilbert (E-H) action of GTR.

Among the various modifications, *f(R)*, *f(T)*, *f(G)*, *f(R, G)* and *f(R, T)* are the mostly studied modified theories of gravity, where *R*, *T* and *G* are respectively the Ricci scalar, Gauss-Bonnet term, and trace of the energy-momentum tensor. The *f(R)* theory of gravity proposed by Buchdahl [45] is treated as the most significant due to its cosmological importance. One of the modifications of the teleparallel equivalent of GTR is the *f(T)* gravity theory. Recently, Pawar et al. [46, 47], studied the accelerating cosmological expansion of the universe in an anisotropic Bianchi type-VI_0 string cosmological model with perfect fluid, and Bianchi type-V cosmological model with dark matter (DM) and HDE, in *f(T)* gravity. Here, we are interested in the *f(R, T)* theory of gravity which has been formulated by Harko et al.[48] in order to explain the present scenario of accelerating cosmic expansion. The *f(R, T)* theory of gravity is a generalization of *f(R)* theory, in which the gravitational Lagrangian in the action of GTR is replaced by a function of *R* and *T*. Many researchers have investigated an accelerated cosmic expansion by studying various cosmological models in *f(R, T)* gravity by taking different kind of matter sources and different volumetric expansion laws. Many researchers have executed impressive work on various cosmological models in *f(R, T)* gravity.

Motivated by the discussions and investigations made by the researchers in past, in this paper we consider an anisotropic Bianchi type-V space-time with perfect fluid in the framework of *f(R, T)* gravity, because certain amount of anisotropy has been observed experimentally, and this space-time describes homogeneity and isotropy which play a decisive role in the early evolution stage of the universe. We construct and study the cosmological models in two different cases: (i) the expansion scalar of the space-time is proportional to the shear scalar (Collins et al., 1980) which gives a relationship between metric potentials and (ii) the law of variation of Hubble's parameter proposed by Berman (1983) which yields the constant deceleration parameter. Also, we study some physical and kinematical parameters and energy conditions for both the models with their graphical behavior. This paper is organized as follows: Section 2 contains the brief review of the work done by some researchers in the framework of *f(R, T)* gravity which is related for the study carried out in this paper, Section 3 provides the brief methodology of *f(R, T)* gravity, Section 4 deals with the metric and derivation of its field equations, Section 5 contain the exact solutions of field equations in two different cases and construction of corresponding cosmological models, Section 6 contain the derivation of some physical and kinematical parameters with the discussion on their graphical behavior, Section 7 deals with the energy conditions of the models, and finally in Section 8, the results and conclusions are presented.

2. Related Work

Sharif and Zubair [49, 50] have studied the behavior of perfect fluid and massless scalar field for homogeneous and anisotropic Bianchi type I universe by assuming the variation law of mean Hubble parameter. They found that their perfect fluid solution can behave like phantom model and correspond to massless scalar field models. Reddy *et al.* [51, 52] have investigated Kaluza-Klein cosmological model in the presence of perfect fluid source and Bianchi type-III cosmological model using the law of variation for the Hubble parameter proposed by Berman [53]. Adhav [54] has studied the LRS Bianchi type-I cosmological model with perfect fluid. Chandel and Ram [55] have studied spatially homogeneous and anisotropic Bianchi type-III space– time in presence of a perfect fluid. They generated new classes of Bianchi type-III cosmological models using the solution of Reddy and co-workers. Samanta [56] has derived exact solutions of the field equations in respect of Kantowski-Sachs universe filled with perfect fluid, and discussed some important features of astrophysical phenomena. Rao and Neelima [57, 58] have obtained perfect fluid Einstein-Rosen and Bianchi type-VI_0 models. Samanta [59] has investigated Bianchi type-V Universe filled with wet dark fluid in both exponential and power-law volumetric expansion, and by using equation of state $p = w(\rho - \rho*)$ for dark energy. Rao et al. [60] have studied spatially homogeneous and anisotropic Bianchi type-II, -VIII and -IX cosmological models filled with perfect fluid and shown that constructed models are anisotropic, expanding, non-rotating and also accelerating. They also noticed that the involvement of the new function $f(R, T)$ does not affect the geometry of the space-time but slightly changes the matter distribution. Yadav [61] constructed Bianchi type V string cosmological model using power law expansion and observed that the massive strings dominate the early universe but they do not survive for long time and finally disappear from the universe. Mishra *et al.* [62] have investigated Bianchi type VI_h cosmological model with perfect fluid. They have also investigated the five-dimensional Kaluza-Klein space time with wet dark fluid (WDF) and they observed the accelerated expansion of the universe [63]. Sahoo *et al.* [64] have studied an axially symmetric cosmological model with perfect fluid and observed that the constructed models represent the accelerated expansion of the universe. Ahmed *et al.* [65] have investigated Bianchi type V cosmological model by considering time dependent deceleration parameter. They considered the cosmological constant Λ as a function of T and dubbed this model as "$\Lambda(T)$ gravity". Rao & Rao [66] have investigated a five dimensional Kaluza-Klein space-time in the presence of anisotropic dark energy by using the special law of variation for Hubble's parameter given by Berman. Sahoo *et al.* [67] have found the big rip situation for LRS Bianchi type I cosmological model that can not be avoided and may be inherent in the linearly varying deceleration parameter. Katore *et al.* [68] have presented the analysis of Bianchi type II, VIII and IX space-time with domain walls. Sahoo *et al*. [69] have investigated an anisotropic Bianchi type-III universe in the presence of a perfect fluid by considering two cases: $f(R, T)=R + 2 f(T)$ and $f(R, T) = f_1(R) + f_2(T)$, and shown that the field equations are solvable for any arbitrary function of a scale factor, and represent an expanding model of the universe which starts expanding with a big bang at $t = 0$. Singh *et al.* [70] have investigated Bianchi type III cosmological model in the presence of cosmological in two cases: $f(R, T) = R + f(T)$ and $f(R, T) = f_1(R) + f_2(T)$ by using the relation that expansion scalar is proportional to shear scalar. Moraes [71] has presented a generalization of this gravity theory by allowing the speed of light to vary, and shown the accelerating expansion of the matter dominated universe, and radiation-dominated universe as a possible alternative to inflationary scenario. Sofuoglu [72] has constructed the functional form of *f(R, T)=R + 2 f(T)* by considering the homogeneous shear-free, rotating and expanding Bianchi type IX universe in the presence of perfect fluid. Sahoo [73] has studied the LRS Bianchi type-I cosmological model in the presence of one dimensional cosmic strings, and Sahoo *et al.* [74] have studied Bianchi-III and - VI_0 cosmological models with string fluid source in the context of late time accelerating expansion of the universe. Samanta and Myrzakulov [75] have studied homogeneous and isotropic universe with bulk viscosity and discussed the effects of bulk viscosity in explaining the early and late time acceleration of the universe. Agrawal and Pawar [76] have studied plane symmetric cosmological model in the presence of quark and strange quark matter and they found that the model does not approach isotropy. Agrawal and Pawar [77] have studied the Bianchi type-V Universe with magnetic domain wall and they observed that the Universe is expanding endlessly under the influence of dark energy. Rashid Zia *et al.* [78] have studied spatially homogeneous and anisotropic Bianchi type-VI_0 dark energy cosmological transit models with string fluid source in the context of early time decelerating and late time accelerating expansion of the universe by using generalized hybrid expansion law. They observed the accelerating expansion of the universe and a phase transition property from decelerating to accelerating. Aditya & Reddy [79] have investigated spatially homogeneous and totally anisotropic Bianchi type-III perfect fluid cosmological model in the presence of an attractive massive scalar field, and obtained a cosmological model of the universe with variable deceleration parameter. Pawar *et al.* [80] have studied the Bianchi type-V cosmological model with modified Holographic Ricci Dark Energy and found that the Universe is in the accelerated expansion phase and it is isotropic throughout the evolution. Also, Pawar *et al.* [81] have studied the behavior of the universe by considering LRS Bianchi-V space–time filled with perfect fluid with heat conduction by using the law of variation for deceleration parameter, and observed that the universe begins with zero volume and large heat flow having infinite expansion rate. Recently, Brahma and Dewri [82] have studied the nature of bulk viscous Bianchi type-V cosmological model in Lyra geometry by considering time dependent displacement field, and found that there is a presence of dark energy, the universe is anisotropic and expanding, and approaches ΛCDM model.

3. A Brief Methodology of *f(R, T)* Gravity

The field equations of *f(R, T)* gravity are obtained from the Hilbert- Einstein type action. The general action of *f(R, T)* gravity given by Harko *et al.* [48] is

$$S=\int [\frac{1}{16\pi G}\, f(R,T)+L_m]\sqrt{-g}\; d^4x\,, \tag{1}$$

where *f(R, T)* is an arbitrary function of the scalar curvature *R* and the trace *T* of the energy-momentum tensor $T_{\alpha\beta}$, and L_m is the matter Lagrangian density, and *g* is the determinant of the metric tensor $g_{\alpha\beta}$. The energy-momentum tensor $T_{\alpha\beta}$ of the matter source is given by

$$T_{\alpha\beta}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,L_m)}{\delta g^{\alpha\beta}} \tag{2}$$

and its trace, $T=g^{\alpha\beta}T_{\alpha\beta}$.

Using gravitational units ($8\pi G$ = c = 1) and by varying the action (1) with respect to the metric tensor $g_{\alpha\beta}$, the field equations of *f(R, T)* gravity are obtained as

$$f_R(R,T)\,R_{\alpha\beta}-\frac{1}{2}f(R,T)\,g_{\alpha\beta}+g_{\alpha\beta}\,(\nabla^{\alpha}\nabla_{\alpha}f_R(R,T))$$
$$-\nabla_{\alpha}\nabla_{\beta}f_R(R,T)=T_{\alpha\beta}-f_T(R,T)T_{\alpha\beta}-f_T(R,T)\,\Theta_{\alpha\beta} \tag{3}$$

where $f_R(R,T)=\frac{\partial f(R,T)}{\partial R}$, $f_T(R,T)=\frac{\partial f(R,T)}{\partial T}$, ∇_{α} denotes the covariant derivative, and $\Theta_{\alpha\beta}$ is defined by

$$\Theta_{\alpha\beta}=-2T_{\alpha\beta}-p\;g_{\alpha\beta} \tag{4}$$

Generally, the field equations depend on the physical nature of the matter field, through the tensor $\Theta_{\alpha\beta}$, therefore, different forms of matter distributions will yield different theoretical models of *f(R, T)* gravity. However, Harko et al. [48] have obtained three particular classes of *f(R, T)* gravity models as

$$f(R,T)=\begin{cases}R+2\;f_1(T)\\ f_1(R)+f_2(T)\\ f_1(R)+f_2(R)\;f_3(T)\end{cases} \tag{5}$$

In the present work, we take $f(R,T)=R+2\,f_1(T)$ with $f_1(T)=\gamma T$, where γ is the coupling constant of *f(R, T)* gravity. This choice of *f(R, T)* together with Eq.(4), reduces the field Eq.(3) in the form:

$$R_{\alpha\beta}-\frac{1}{2}R\;g_{\alpha\beta}=(1+2\gamma)\,T_{\alpha\beta}+\gamma\,(2p+T)\,g_{\alpha\beta} \tag{6}$$

4. Metric and Field Equations

We consider an anisotropic Bianchi type-V space-time in the form:

$$ds^2=dt^2-X^2\,dx^2-e^{2mx}\,(Y^2\,dy^2+Z^2\,dz^2) \tag{7}$$

where *X, Y, Z* are functions of cosmic time *t* only, and *m* is a constant.

Here, we consider the matter source as a perfect fluid having energy-momentum tensor as given by

$$T_{\alpha\beta}=(\rho+p)u_{\alpha}u_{\beta}-p\;g_{\alpha\beta}\,, \tag{8}$$

where ρ and p are the energy density and equilibrium pressure of the fluid respectively.

In co-moving coordinates, $u^{\alpha}=(0,0,0,1)$, where u^{α} is the four velocity of the fluid satisfying the condition $u_{\alpha}u^{\alpha}=1$.

Thus, from Eq. (8), the components of energy-momentum tensor are obtained as

$$T_1^1 = T_2^2 = T_3^3 = -p,\ T_4^4 = \rho, \tag{9}$$

and hence $T = \rho - 3p$.

The Ricci scalar for the metric (7) is obtained as

$$R = 2\left\{\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{X}\dot{Z}}{XZ} + \frac{\dot{Y}\dot{Z}}{YZ} - \frac{3m^2}{X^2}\right\} \tag{10}$$

Using the co-moving coordinate system, the field equation (6) with the help of (8) for the metric (7) yields the equations:

$$\frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{Y}\dot{Z}}{YZ} - \frac{m^2}{X^2} = (1+3\gamma)\,p - \gamma\rho, \tag{11}$$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Z}}{Z} + \frac{\dot{X}\dot{Z}}{XZ} - \frac{m^2}{X^2} = (1+3\gamma)\,p - \gamma\rho, \tag{12}$$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\dot{X}\dot{Y}}{XY} - \frac{m^2}{X^2} = (1+3\gamma)\,p - \gamma\rho, \tag{13}$$

$$\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{X}\dot{Z}}{XZ} + \frac{\dot{Y}\dot{Z}}{YZ} - \frac{3m^2}{X^2} = \gamma p - (1+3\gamma)\rho, \tag{14}$$

$$2\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} - \frac{\dot{Z}}{Z} = 0. \tag{15}$$

Here an overhead dot denotes the ordinary derivative with respect to *t*.

5. Solutions of the Field Equations

Integrating Eq. (15), we get

$$X^2 = hYZ,$$

where h is a constant of integration. Without loss of generality, we choose the constant h =1, then above equation becomes

$$X^2 = YZ \tag{16}$$

Solving Eqs. (11) – (16), we obtain

$$Y = C_1 X\, e^{\frac{k}{2}\int \frac{dt}{X^3}}, \tag{17}$$

$$Z = C_2 X\, e^{\frac{-k}{2}\int \frac{dt}{X^3}}, \tag{18}$$

$$2\frac{\ddot{X}}{X} + \frac{\dot{X}^2}{X^2} + \frac{k^2}{4X^6} - \frac{m^2}{X^2} = (1+3\gamma)\,p - \gamma\rho, \tag{19}$$

$$3\frac{\dot{X}^2}{X^2} - \frac{k^2}{4X^6} - \frac{3m^2}{X^2} = \gamma p - (1+3\gamma)\rho, \tag{20}$$

where k, C_1, C_2 are constants of integration such that $C_1 C_2 = 1$.

The spatial volume (V) for the space-time (7) is given by $V = XYZ$. (21)

Eq. (21), together with (16), yields the average scale factor, $a = \sqrt[3]{V} = X$. (22)

Some other cosmologically important physical and kinematical parameters for the space-time (7) are defined as follows:

The average Hubble's parameter,

$$H = \frac{1}{3}\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right) = \frac{\dot{a}}{a}. \tag{23}$$

The expansion scalar and shear scalar are respectively,

$$\theta = 3H = 3\frac{\dot{a}}{a}, \tag{24}$$

and

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{X}}{X}\right)^2 + \left(\frac{\dot{Y}}{Y}\right)^2 + \left(\frac{\dot{Z}}{Z}\right)^2\right] - \frac{\theta^2}{6} = \frac{k^2}{4a^6}. \tag{25}$$

The average anisotropy parameter,

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{k^2}{6\dot{a}^2 a^4}. \tag{26}$$

The Deceleration parameter,

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{27}$$

Solving Eqs. (19) and (20), and using (22), we obtain the energy density ρ and pressure *p* as follows:

$$\rho = \frac{1}{(1+2\gamma)(1+4\gamma)}\left[2\gamma\left(\frac{\ddot{a}}{a}\right) - (3+8\gamma)\left\{\left(\frac{\dot{a}}{a}\right)^2 - \left(\frac{m}{a}\right)^2\right\}\right] + \frac{k^2}{4(1+2\gamma)\,a^6} \tag{28}$$

$$\rho = \frac{1}{(1+2\gamma)(1+4\gamma)}\left[2(1+3\gamma)\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 - \left(\frac{m}{a}\right)^2\right] + \frac{k^2}{4(1+2\gamma)\,a^6} \tag{29}$$

Further we solve the field equations in two different cases and accordingly we obtain two different cosmological models.

5.1 Model 1:

We assume that the shear scalar σ is proportional to the expansion scalar θ [83], which leads to

$$Y = Z^n \tag{30}$$

where *n* is an arbitrary constant such that the model is of anisotropic nature for $n \neq 1$.

Eqs. (12), (13) and (15), together with (30), yield

$$X = \left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t + \alpha\right]^{1/3}, \tag{31}$$

$$Y = \left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t + \alpha\right]^{2n/3(n+1)}, \tag{32}$$

$$Z = \left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t + \alpha\right]^{2/3(n+1)}. \tag{33}$$

where α is a constant of integration.

Using Eqs. (31) - (33) in (7), we obtain the Bianchi type-V perfect fluid cosmological model in *f(R, T)* gravity in the form:

$$ds^2 = dt^2 - \left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t + \alpha\right]^{\frac{2}{3}} dx^2 - e^{2mx}\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t + \alpha\right]^{\frac{4n}{3(n+1)}} dy^2 - e^{2mx}\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t + \alpha\right]^{\frac{4}{3(n+1)}} dz^2 \tag{34}$$

This model is free from the singularities for $n \neq 1$.

5.2 Model 2:

We consider the commonly used law of variation of Hubble's parameter which yields the constant value of deceleration parameter given by [53],

$$H=\beta a^{-l} \tag{35}$$

where $\beta>0$ and $l\geq 0$ are constants.

From Eqs. (22), (23) and (35), we obtain

$$X=(l\beta t)^{1/l} \tag{36}$$

Eqs. (17) and (18) with the use of (36) yield

$$Y=C_1(l\beta t)^{\frac{1}{l}}\ e^{\frac{k}{2\beta(l-3)}(l\beta t)^{\frac{l-3}{l}}}, \tag{37}$$

$$Z=C_2(l\beta t)^{\frac{1}{l}}\ e^{\frac{-k}{2\beta(l-3)}(l\beta t)^{\frac{l-3}{l}}}. \tag{38}$$

Using Eqs. (36) - (38) in (7), we obtain the Bianchi type-V perfect fluid cosmological model in *f(R, T)* gravity in the form:

$$ds^2=dt^2-(l\beta t)^{2/l}\,dx^2-(l\beta t)^{2/l}\ e^{2mx}\left[C_1^{\,2}\,e^{\frac{k}{\beta(l-3)}(l\beta t)^{\frac{l-3}{l}}}\,dy^2+C_2^{\,2}\,e^{\frac{-k}{\beta(l-3)}(l\beta t)^{\frac{l-3}{l}}}\,dz^2\right] \tag{39}$$

This model is singularity free for $l\neq 0,3$.

6. Some Physical and Kinematical Properties of the Models

Some physical and kinematical parameters of the derived cosmological models, namely, spatial volume (V), scale factor (a), the Hubble's parameter (H), expansion scalar (θ), shear scalar (σ^2) anisotropy parameter (A_m), deceleration parameter (q), the energy density (ρ) and the pressure (p), which play an important role in the physical discussion of the model, are obtained in this section.

Model 1: From Eqs. (21) – (29) with the use of (31) – (33), we obtain the following:

$$V=\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha, \tag{40}$$

$$a=\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha\right]^{1/3}, \tag{41}$$

$$H=\frac{k}{2}\left(\frac{n+1}{n-1}\right)\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha\right]^{-1}, \tag{42}$$

$$\theta=\frac{3k}{2}\left(\frac{n+1}{n-1}\right)\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha\right]^{-1}, \tag{43}$$

$$\sigma^2=\frac{k^2}{4}\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha\right]^{-2}, \tag{44}$$

$$A_m=\frac{2}{3}\left(\frac{n-1}{n+1}\right)^2, \tag{45}$$

$$q=2, \tag{46}$$

$$\rho=\frac{k^2}{4(1+2\gamma)}\left[1-3\left(\frac{n+1}{n-1}\right)^2\right]\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha\right]^{-2}+\frac{(3+8\gamma)m^2}{(1+2\gamma)(1+4\gamma)}\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha\right]^{-2/3}, \tag{47}$$

$$p=\frac{k^2}{4(1+2\gamma)}\left[1-3\left(\frac{n+1}{n-1}\right)^2\right]\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha\right]^{-2}-\frac{m^2}{(1+2\gamma)(1+4\gamma)}\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha\right]^{-2/3}. \tag{48}$$

From (43) and (44), we obtain

$$\frac{\sigma}{\theta}=\frac{1}{3}\left(\frac{n-1}{n+1}\right). \tag{49}$$

From the above results, it can be observed that, the spatial volume is constant at t = 0 and increases linearly with time, i.e., the universe expands linearly.

The parameters V, a, H, θ, σ^2, A_m, ρ and p are all constant at t = 0, $n\neq 1$; shows that the universe starts to evolve with constant volume and expands exponentially. Also, H, θ, σ^2, ρ and p vanish as $t\to\infty$.

The value of A_m shows that the universe is anisotropic for all $n\neq 1$. Also, the condition stated by Collins and Hawking [84], i.e., $\lim_{t\to\infty}\frac{\sigma}{\theta}=0$, does not hold for $n\neq 1$, and hence the model dose not approach isotropy throughout the evolution of the universe.

Here, the deceleration parameter q = 2 shows that the universe decelerates in a standard way, which is not in accordance with the present scenario of accelerating universe. This result is similar to the results discussed by Huang *et al.* [85], Reddy *et al.* [51] and Rao *et al*. [86], respectively, in the study of five dimensional *f(R)* gravity, Kaluza-Klein cosmological model in *f(R, T)* gravity and Bianchi type-I cosmological model with perfect fluid in *f(R, T)* gravity.

We have discussed below the graphical behavior of some parameters for model 1. We fixed the constants as $k=\gamma=1$, $\alpha=m=2$ in order to get positive energy density.

The variation of energy density (ρ) with time is depicted in Figure 1 for n = 1.4, 1.6, 1.8. It is observed that for $n\geq 1.5$ the value of (ρ) is positive and finite at early time, and it decreases with time and approaches to zero in the late time. For $n<1.5$, it starts with negative value, increases rapidly and becomes positive, and then behave as in case for $n\geq 1.5$.

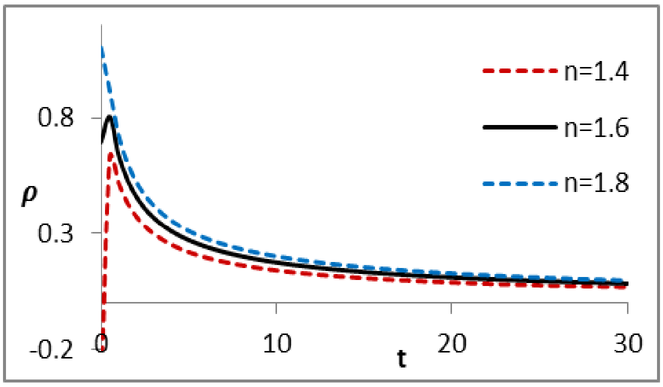

Figure 1: Plot of energy density (ρ) vs. cosmic time *t* for model 1, with $k = \gamma = 1,\ \alpha = m = 2$.

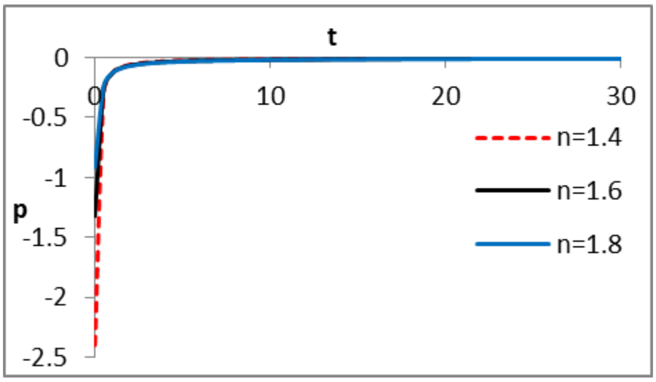

Figure 2: Plot of the pressure (*p*) vs. cosmic time *t* for model 1, with $k = \gamma = 1,\ \alpha = m = 2$.

Figure 2 depicts the variation of matter pressure *p* with time *t*. It is noticed that $p < 0$ throughout the evolution of the universe, and hence perfect fluid behave like the dark energy of phantom type.

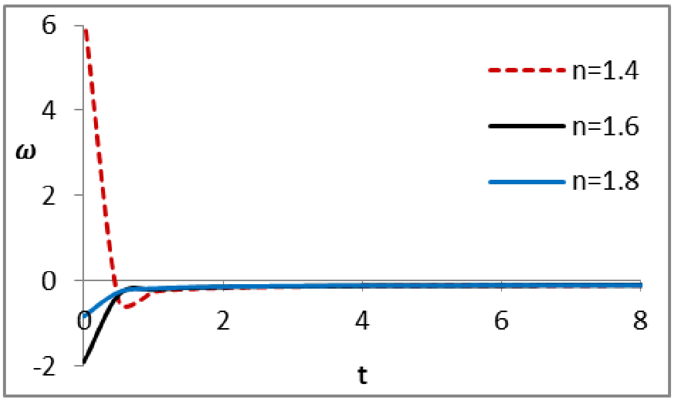

Figure 3: Plot of EoS parameter ω vs. cosmic time *t* for model 1, with $k = \gamma = 1,\ \alpha = m = 2$.

The graphical behavior of EoS parameter $\omega (= p/\rho)$ is shown in Figure 3. For $n < 1.5$, it has positive finite value greater than 1 at early time, decreases abruptly and becomes negative, and then increases with time and approaches to a constant negative value (-0.09) in late time. For $n \geq 1.5$, it has negative finite value at early time, increases with time and approaches to the same constant negative value (-0.09) in late time, i.e., $\omega > -1$, and hence the model remain present in quintessence region.

The overall energy density parameter for the cosmic fluid defined by $\Omega = \frac{\rho}{3H^2}$, is obtained as

$$\Omega = \frac{-2(n^2+4n+1)}{3(1+2\gamma)(n+1)^2} + \frac{4\,(3+8\gamma)m^2}{3k^2(1+2\gamma)(1+4\gamma)}\left(\frac{n-1}{n+1}\right)^2\left[\frac{3k}{2}\left(\frac{n+1}{n-1}\right)t+\alpha\right]^{\frac{4}{3}} \quad (50)$$

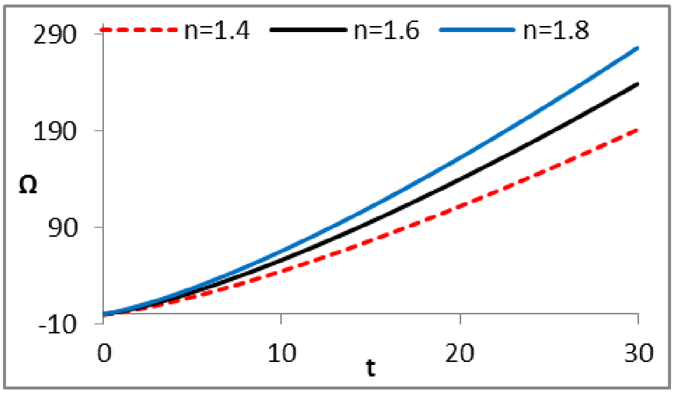

Figure 4: Plot of overall energy density parameter Ω vs. cosmic time *t* for model 1, with $k = \gamma = 1,\ \alpha = m = 2$.

From the graphical behavior of overall energy density parameter Ω depicted in Figure 4, we observe that $\Omega < 1$ for a very short initial period of time and then it increases rapidly with time so that $\Omega > 1$. Thus, according to observational results of SNe Ia and CMB experiments, the behavior of the universe is open initially for a very short period, but then it is closed throughout the evolution.

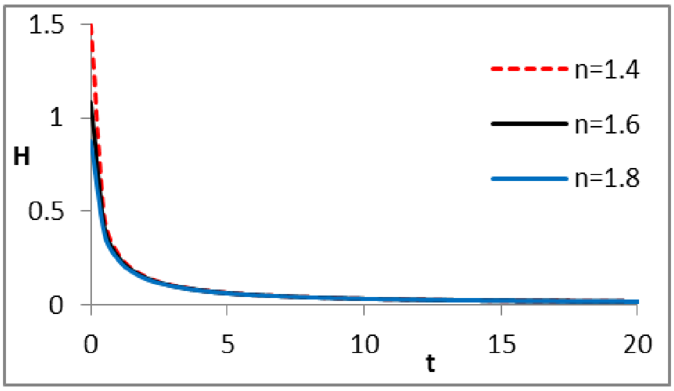

Figure 5: Plot of the Hubble parameter H vs. cosmic time *t* for model 1, with $k = 1,\ \alpha = 2$.

Figure 5 represents the behavior of the Hubble parameter versus cosmic time *t*, in which it is observed that *H* decreases as *t* increases and it remains positive but close to zero in late time. This shows that the universe expands gradually. Also,

$H > 0$ and $q > 0$ shows the decelerating expansion of the universe.

Model-2: From Eqs. (21) – (29) with the use of (36) – (38), we obtain the various parameters as follows:

$$V = (l\beta t)^{\frac{3}{l}}, \tag{51}$$

$$a = (l\beta t)^{\frac{1}{l}}, \tag{52}$$

$$H = \frac{1}{l\,t}, \tag{53}$$

$$\theta = \frac{3}{l\,t}, \tag{54}$$

$$\sigma^2 = \frac{k^2}{4}(l\beta t)^{-\frac{6}{l}}, \tag{55}$$

$$A_m = \frac{k^2}{6\beta^2}(l\beta t)^{\frac{2(l-3)}{l}}, \tag{56}$$

$$q = l - 1, \tag{57}$$

$$\rho = \frac{1}{(1+2\gamma)(1+4\gamma)}\left\{\frac{-2\gamma(l+3)-3}{l^2t^2} + \frac{(3+8\gamma)m^2}{(l\beta t)^{2/l}} + \frac{(1+4\gamma)k^2}{4(l\beta t)^{6/l}}\right\}, \tag{58}$$

$$p = \frac{1}{(1+2\gamma)(1+4\gamma)}\left\{\frac{6\gamma(1-l)-2l+3}{l^2t^2} - \frac{m^2}{(l\beta t)^{2/l}} + \frac{(1+4\gamma)k^2}{4(l\beta t)^{6/l}}\right\} \tag{59}$$

Also, from (54) and (55), we obtain

$$\frac{\sigma}{\theta} = \frac{k}{6\beta}(l\beta t)^{\frac{l-3}{l}}. \tag{60}$$

We observe that the universe starts with a zero volume which increases with time. The parameters H, θ , σ^2 , ρ and p diverge for $t = 0$, and they approach to 0 as $t \to \infty$. As the pressure and density are infinite at $t = 0$, the model has initial singularity.

The constructed model is anisotropic for $l \neq 0$. Also, $\lim_{t\to\infty}\frac{\sigma}{\theta} = 0$ does not hold for $l \neq 0$, and hence the model dose not approach isotropy throughout the evolution of the universe.

It is observed that, under the law of variation of H, the value of deceleration parameter is constant. The model represents decelerating phase of expansion for $l > 1$ and accelerating phase of expansion for $l < 1$. For $l = 1$, we get $q = 0$ and $H = 1/t$, which shows that the universe expands with a constant speed. For $l = 0$, we get $q = -1$ and $H = \beta > 0$ (a constant). The value $q = -1$ represents the accelerating phase of the universe; while the Hubble parameter which is the large scale property of the universe is constant throughout the time imply the steady state model of the universe.

The model represents shearing, non-rotating and expanding universe.

We have discussed the graphical behavior of some parameters by keeping the view of accelerating universe in mind, and accordingly taken the values of a constant l. For accelerating cosmic expansion the observed value of q at present epoch is -0.73 [87], and we obtain it at $l = 0.27$. Hence we take one of the values of l is equal to 0.27 ($l < 1$, for accelerating phase) and other values as 1 (for steady state expansion) and 1.1 ($l > 1$, for decelerating phase) for the discussion of graphical behavior of the parameters. The particular values of constants are chosen as $k = \gamma = 1$, $m = 2$, $\beta = 0.5$ in order to get positive energy density.

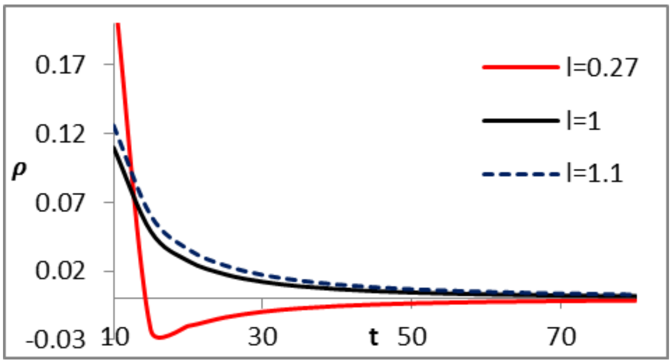

Figure 6: Plot of energy density (ρ) vs. cosmic time t for model 2, with $k = \gamma = 1$, $m = 2$, $\beta = 0.5$.

Graphical behavior of matter energy density (ρ) is depicted in figure 6 for $l = 0.27$, 1, 1.1. It is noticed that, for all these values of l, the energy density is very high at an initial epoch, but at later time it decrease rapidly and tend to zero as $t \to \infty$. Also, for $l = 0.27$, at later time, ρ becomes negative and remains negative throughout the evolution of the universe, which causes the repulsion of ordinary matter. This is closest to the real phenomenon called Casimir effect. In this case, if the universe is open then it will either expand indefinitely or it eventually turns out into a big rip. But for $l = 1$, 1.1, the value of ρ never becomes negative. Thus, we observe that the universe expands with very high rate initially which slower down with time.

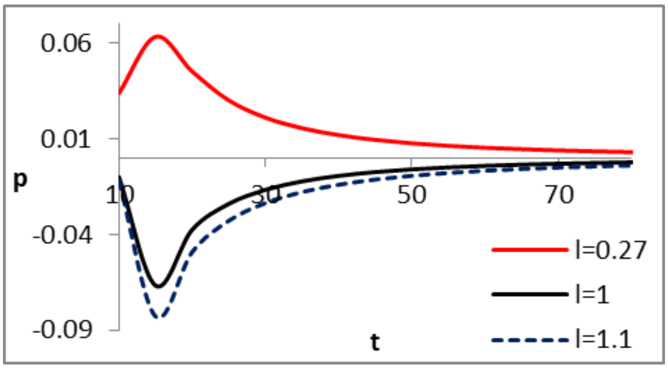

Figure 7: Plot of the pressure p vs. cosmic time t for model 2, with $k = \gamma = 1$, $m = 2$, $\beta = 0.5$.

Figure 7 depicts the variation of matter pressure p with time t for $t \geq 10$ and $l = 0.27$, 1, 1.1. We have observed that for all these values of l, the pressure is very high at an initial epoch, then it decreases rapidly and becomes negative. Then in later time, it increases but remains negative for $l = 1$, 1.1, while remains positive for $l = 0.27$. Hence in later time for $l = 1$, 1.1, perfect fluid behaves like the dark energy of phantom type, which is not the case for l = 0.27.

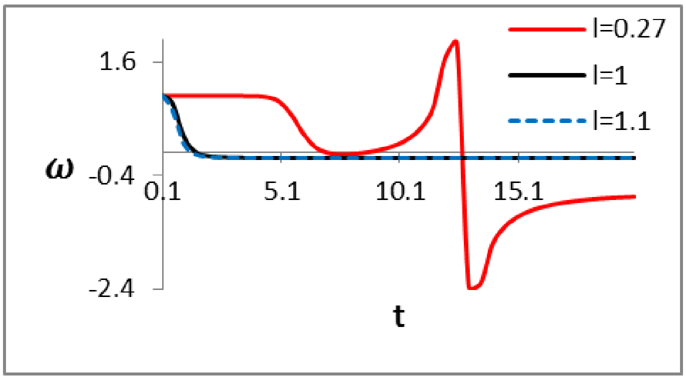

Figure 8: Plot of EoS parameter ω vs. cosmic time t for model 2, with $k=\gamma=1,\ m=2,\ \beta=0.5$.

From the graphical behavior of EoS parameter (ω) depicted in Figure 8, it is observed for l = 0.27 that the value of ω oscillates between positive (=1) to negative (-2.4, approx.) in the early stage of evolution, and later it becomes constant (= -0.71, approx.), i.e., the universe model shifts from matter dominated to phantom in the early stage and then remain present in the quintessence region in later time. For $l\geq 1$, it is observed that the universe is matter dominated in the early stage and then remain present quintessence region in later time.

In this case, the density parameter for the cosmic fluid is obtained as

$$\Omega=\frac{-2\gamma(l+3)-3}{3(1+2\gamma)(1+4\gamma)}+\frac{(3+8\gamma)m^2}{3\beta^2(1+2\gamma)(1+4\gamma)}(l\beta t)^{\frac{2(l-1)}{l}}+\frac{k^2}{12\beta^2(1+2\gamma)}(l\beta t)^{\frac{2(l-3)}{l}} \tag{61}$$

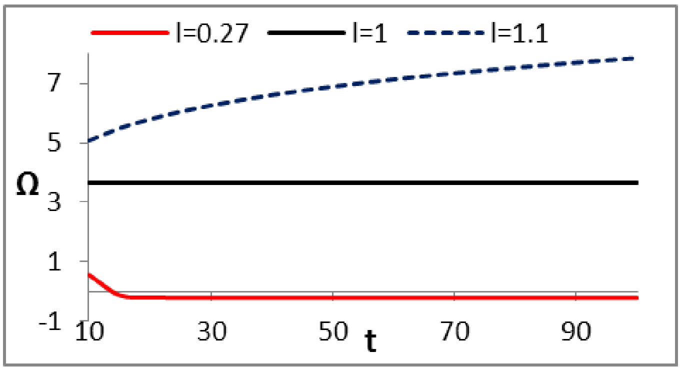

Figure 9: Plot of overall energy density parameter Ω vs. cosmic time t for model 2, with k=γ=1, m=2, $\beta=0.5$.

From the graphical behavior of overall energy density parameter Ω for model-2 is shown in Figure 9. It can be observed for l = 0.27 that, $\Omega>1$ for a very short initial period of time and then it decreases rapidly with time and assumes constant negative value; i.e., the universe is closed at an initial epoch and later it is open. It is observed for $l\geq 1$ that, $\Omega>1$ through the evolution of the universe, and hence it represents the closed model of the universe.

7. Energy conditions

Energy conditions stated in [88] are given by

Null energy conditions (NEC): $\rho+p\geq 0$,

Weak energy conditions (WEC): $\rho\geq 0,\ \rho+p\geq 0$,

Dominant energy conditions (DEC): $\rho\geq 0,\ \rho\pm p\geq 0$,

Strong energy conditions (SEC): $\rho+3p\geq 0$

The plots for energy conditions of the models so constructed are given in the figures below:

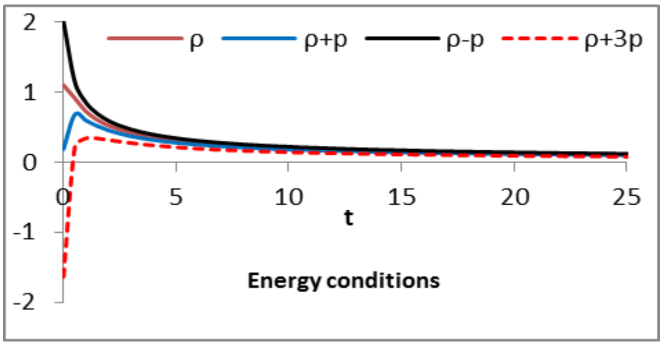

Figure 10: Plots of energy conditions for model 1, with $k=\gamma=1,\ m=2,\ \beta=0.5$.

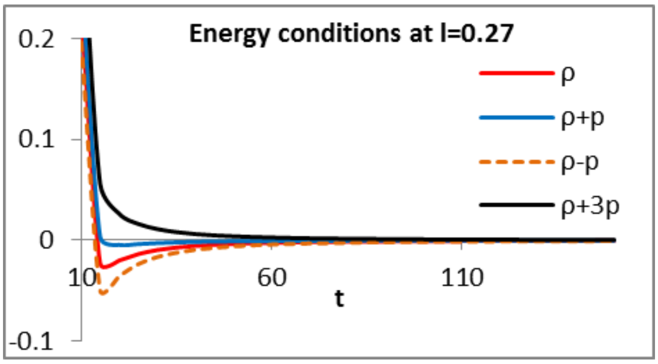

Figure 11(a): Plots of energy conditions for model 2, with l = 0.27, and $k=\gamma=1,\ m=2,\ \beta=0.5$.

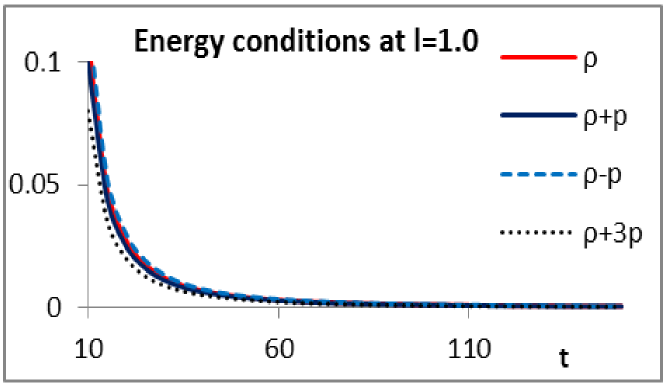

Figure 11(b): Plots of energy conditions for model 2, with l = 1, and $k=\gamma=1,\ m=2,\ \beta=0.5$.

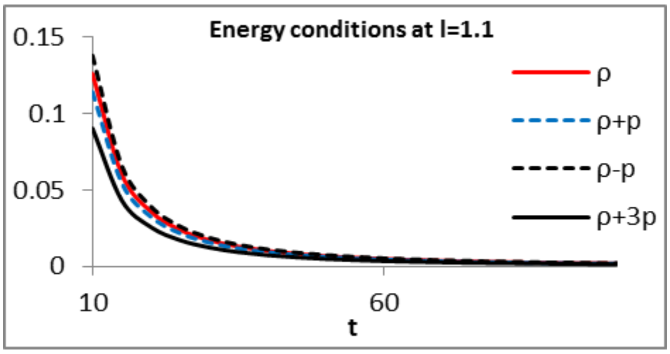

Figure 11(c): Plots of energy conditions for model 2, with l = 1.1, and $k = \gamma = 1$, $m = 2$, $\beta = 0.5$.

From Figure 10, it is observed that all the energy conditions except SEC are satisfied in the model 1. The SEC is also satisfied at late times.

The energy conditions for model 2 are depicted in Figure 11(a, b, c) at l = 0.27, 1, 1.1. It is observed that only SEC is satisfied for model 2 at l = 0.27, while all the energy conditions are satisfied at l = 1 and l = 1.1.

8. Results and Conclusion

Here we have constructed anisotropic Bianchi type-V cosmological models with perfect fluid in the framework of *f(R, T)* gravity proposed by Harko et al. [48]. Two different cosmological models are constructed by solving the field equations in two different cases: (i) the expansion scalar of the space-time is proportional to the shear scalar (Collins et al., [83]) which yields a relationship between metric potentials, taken as $Y = Z^n$; and (ii) the law of variation of Hubble's parameter proposed by Berman [53]: $H = \beta a^{-l}$ (where $\beta > 0$ and $l \geq 0$ are constants), which yields the constant deceleration parameter. The models so constructed are free from singularities for $n \neq 1$ and $l \neq 0, 3$, respectively.

Model 1: It is free from singularities for $n \neq 1$. The values of the parameters *V*, *a*, *H*, θ, σ^2, A_m, ρ and *p* shows that the universe is anisotropic for $n \neq 1$, and evolves with constant volume and expands exponentially. The condition stated by Collins and Hawking [84], i.e., $\lim_{t \to \infty} \frac{\sigma}{\theta} = 0$, does not hold for $n \neq 1$, and hence the model dose not approach isotropy throughout the evolution of the universe. The deceleration parameter *q* = 2 shows that the universe decelerates in a standard way, which is not in accordance with the present scenario of accelerating universe. This result is similar to the results discussed by Huang *et al.* [85], Reddy *et al.* [51] and Rao *et al*. [86], respectively, in the study of five dimensional *f*(*R*) gravity, Kaluza-Klein cosmological model in *f*(*R*, *T*) gravity and Bianchi type-I cosmological model with perfect fluid in *f*(*R*, *T*) gravity. The matter energy density is positive and a decreasing function of time. As the matter pressure is negative throughout the evolution of the universe, the perfect fluid behaves like a dark energy of phantom type. It is observed from the graph of EoS parameter ω that the universe is matter dominated ($\omega > 0$) at the initial epoch and it remain present in the quintessence region ($\omega > -1$) at late times. It is observed that the overall energy $\Omega < 1$ for a very short initial period of time and then it increases rapidly with time so that $\Omega > 1$; and hence, according to observational results of SNe Ia and CMB experiments, the behavior of the universe is open initially for a very short period, but then it is closed throughout the evolution. The Hubble's parameter *H* is a decreasing function of time and it remains positive but close to zero at late times, and *H* > 0 and *q* > 0, shows the decelerating expansion of the universe. All the energy conditions except SEC are satisfied in this model 1, and at late times SEC is also satisfied. Overall, the derived model resembles with the existing models with perfect fluid, particularly with the perfect fluid models studied by Reddy *et al.* [51] and Rao *et al*. [86].

Model 2: It is free from singularities for $l \neq 0, 3$. But the pressure and density are infinite at *t* = 0, the model has initial singularity. The universe starts with a zero volume which increases with time. The model is anisotropic for $l \neq 0$. Also, $\lim_{t \to \infty} \frac{\sigma}{\theta} = 0$ does not hold for $l \neq 0$, and hence the model dose not approach isotropy throughout the evolution of the universe. It is observed that, under the law of variation of *H*, the value of deceleration parameter is constant. The model represents decelerating phase of expansion for l > 1, and accelerating phase of expansion for l < 1 and l = 0, and expansion with constant rate for l = 1. For accelerating cosmic expansion the observed value of *q* at present epoch is -0.73 [87], and it is at l = 0.27. Hence the behavior of cosmological parameters are discussed graphically at l = 0.27 (l < 1, for accelerating phase), l = 1(for steady state expansion) and l = 1.1 (l > 1, for decelerating phase). For all these assumed values of l, it is observed that the universe expands with very high rate initially which slower down with time; and for l = 1, 1.1, the perfect fluid behaves like a dark energy of phantom type. Also, for l = 0.27, at later time, ρ becomes negative and remains negative throughout the evolution of the universe, which causes the repulsion of ordinary matter. This is closest to the real phenomenon called Casimir effect. In this case, if the universe is open then it will either expand indefinitely or it eventually turns out into a big rip. Graphical behavior of EoS parameter shows that the universe is matter dominated ($\omega > 0$) in the early stage and then remain present in quintessence region ($\omega > -1$) at later times. For l = 0.27, the universe is closed ($\Omega > 1$) at an initial epoch and at later times it is open ($\Omega < 1$), while for $l \geq 1$ it is closed throughout the passage of time. Only the SEC is satisfied for model 2 at l = 0.27, while all the energy conditions are satisfied at l = 1 and l = 1.1.

Thus, the results obtained in case of both the constructed models are compatible with the existing observations, and hence the models are physically acceptable.

Conflict of Interest
Authors declare that they do not have any conflict of interest.

Funding Source
This research work has been carried out without any support from funding agencies.

Acknowledgements
Authors are grateful to the reviewers and editors for their constructive comments and suggestions which have helped in the improvement of the manuscript.

References

[1] P. M. Garnavich, P. K. Robert, P. Challis, J. Tonry, R. L. Gilliland, R. Chris Smith, A. Clocchiatti et al., "Constraints on cosmological models from Hubble Space Telescope observations of high-z supernovae", *The Astrophysical Journal*, Vol. **493**, No. **2**, p. **53**, **1998**.

[2] P. M. Garnavich, S. Jha, P. Challis, A. Clocchiatti, A. Diercks, A. V. Filippenko, R. L. Gilliland et al., "Supernova limits on the cosmic equation of state", *The Astrophysical Journal*, Vol. **509**, No. **1,** p. **74**, **1998**.

[3] A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland et al., "Observational evidence from supernovae for an accelerating universe and a cosmological constant", *The astronomical journal*, Vol. **116**, No. **3,** p. **1009**, **1998**.

[4] A. G. Riess, L. G. Strolger, J. Tonry, S. Casertano, H. C. Ferguson, B. Mobasher, P. Challis et al., "Type Ia supernova discoveries at z> 1 from the Hubble Space Telescope: Evidence for past deceleration and constraints on dark energy evolution", *The Astrophysical Journal*, Vol. **607**, No. **2**, p. **665**, **2004**.

[5] S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua et al., "Measurements of Ω and Λ from 42 high-redshift supernovae", *The Astrophysical Journal*, Vol. **517**, No. **2**, p. **565**, **1999**.

[6] C. L. Bennett, "First year Wilkinson microwave anisotropy probe (WMAP) observations: Preliminary maps and basic results", *Astrophys. J. Suppl*, Vol. **148**, No. **1, 2003**.

[7] D. N. Spergel, L. Verde, H. V. Peiris, E. Komatsu, M. R. Nolta, C. L. Bennett, M. Halpern et al., "First-year Wilkinson Microwave Anisotropy Probe (WMAP)* observations: determination of cosmological parameters", *The Astrophysical Journal Supplement Series*, Vol. **148**, No. **1**, p. **175**, **2003**.

[8] R. R. Caldwell, and M. Doran, "Cosmic microwave background and supernova constraints on quintessence: concordance regions and target models", *Physical Review D*, Vol. **69**, No. **10**, p. **103517**, **2004**.

[9] Z. Y. Huang, B. Wang, E. Abdalla, and R. K. Su, "Holographic explanation of wide-angle power correlation suppression in the cosmic microwave background radiation", *Journal of Cosmology and Astroparticle Physics*, Vol. **2006**, No. **05**, p. **013**, **2006**.

[10] M. Tegmark, M. A. Strauss, M. R. Blanton, K. Abazajian, S. Dodelson, H. Sandvik, X. Wang et al., "Cosmological parameters from SDSS and WMAP", *Physical review D*, Vol. **69**, No. **10**, p. **103501**, **2004**.

[11] M. Tegmark, M. R. Blanton, M. A. Strauss, F. Hoyle, D. Schlegel, R. Scoccimarro, M. S. Vogeley et al., "The three-dimensional power spectrum of galaxies from the sloan digital sky survey", *The Astrophysical Journal*, Vol. **606**, No. **2**, p. **702**, **2004**.

[12] D. J. Eisenstein, I. Zehavi, D. W. Hogg, R. Scoccimarro, M. R. Blanton, R. C. Nichol, R. Scranton et al., "Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies", *The Astrophysical Journal*, Vol. **633**, No. **2**, p. **560**, **2005**.

[13] S. Weinberg, "The cosmological constant problem", *Reviews of modern physics*, Vol. **61**, No. **1**, p. **1**, **1989**.

[14] S. M. Carroll, "The cosmological constant", *Living reviews in relativity*, Vol. **4**, No. **1**, pp. **1-56**, **2001**.

[15] P. J. E. Peebles, and Bharat Ratra, "The cosmological constant and dark energy", *Reviews of modern physics*, Vol. **75**, No. **2**, p. **559**, **2003**.

[16] T. Padmanabhan, "Cosmological constant—the weight of the vacuum", *Physics Reports*, Vol. **380**, No. **5-6,** pp. **235-320**, **2003**.

[17] S. Nojiri, and S. D. Odintsov, "Introduction to modified gravity and gravitational alternative for dark energy", *International Journal of Geometric Methods in Modern Physics*, Vol. **4**, No. **01**, pp. **115-145**, **2007**.

[18] N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A. J. Banday et al., "Planck 2018 results-VI. Cosmological parameters", *Astronomy & Astrophysics*, Vol. **641,** p. **A6, 2020**.

[19] C. Wetterich, "Cosmology and the fate of dilatation symmetry", *Nuclear Physics B*, Vol. **302**, No. **4**, pp. **668-696**, **1988**.

[20] B. Ratra, and P. J. E Peebles, "Cosmological consequences of a rolling homogeneous scalar field", *Physical Review D*, Vol. **37**, No. **12**, p. **3406**, **1988**.

[21] M. Cicoli, F. G. Pedro, and G. Tasinato, "Natural quintessence in string theory", *Journal of Cosmology and Astroparticle Physics*, Vol. **2012**, No. **07**, p. **044**, **2012**.

[22] C. Wetterich, "Inflation, quintessence, and the origin of mass", *Nuclear Physics B*, Vol. **897**, pp. **111-178**, **2015**.

[23] S. Tsujikawa, "Quintessence: a review", *Classical and Quantum Gravity*, Vol. **30**, No. **21**, p. **214003**, **2013**.

[24] C. Armendariz-Picon, T. Damour, and V. Mukhanov, "k-Inflation", *Physics Letters B*, Vol. **458**, No. **2-3**, pp. **209-218**, **1999**.

[25] T. Chiba, T. Okabe, and M. Yamaguchi, "Kinetically driven quintessence", *Physical Review D*, Vol. **62**, No. **2**, p. **023511**, **2000**.

[26] G. Varshney, U. K. Sharma, and A. Pradhan, "Reconstructing the k-essence and the dilation field models of the THDE in f (R, T) gravity", *The European Physical Journal Plus*, Vol. **135**, No. **7**, p. **541**, **2020.**

[27] R. R. Caldwell, "A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state", *Physics Letters B*, Vol. **545**, No. **1-2**, pp. **23-29**, **2002**.

[28] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, "Phantom energy: dark energy with w<− 1 causes a cosmic doomsday", *Physical review letters*, Vol. **91**, No. **7**, p. **071301**, **2003**.

[29] E. Elizalde, S. Nojiri, and S. D. Odintsov, "Late-time cosmology in a (phantom) scalar-tensor theory: dark energy and the cosmic speed-up", *Physical Review D*, Vol. **70**, No. **4**, p. **043539**, **2004.**

[30] A. Anisimov, E. Babichev, and A. Vikman, "B-inflation", *Journal of Cosmology and Astroparticle Physics*, Vol. **2005**, No. **06**, p. **006**, **2005**.

[31] A. Sen, "Rolling tachyon", *Journal of High Energy Physics*, Vol. **2002**, No. **04,** p. **048, 2002**.

[32] T. Padmanabhan, and T. Roy Choudhury, "Can the clustered dark matter and the smooth dark energy arise from the same scalar field?", *Physical Review D*, Vol. **66**, No. **8**, p. **081301**, **2002**..

[33] J. Khoury, and A. Weltman, "Chameleon fields: Awaiting surprises for tests of gravity in space", *Physical review letters*, Vol. **93**, No. **17**, p. **171104**, **2004**..

[34] S. Nojiri, S. D. Odintsov, V. K. Oikonomou, and T. Paul, "Unifying holographic inflation with holographic dark energy: A covariant approach", *Physical Review D*, Vol. **102**, No. **2**, p. **023540**, **2020**..

[35] S. H. Shekh, "Models of holographic dark energy in f (Q) gravity", *Physics of the dark Universe*, Vol. **33,** p. **100850**, **2021**.

[36] S.Wang, Y. Wang, and M. Li, "Holographic dark energy", *Physics reports*, Vol. **696**, pp. **1-57**, **2017**.

[37] S. D. Katore, S. V. Gore, and A. Y. Shaikh, "Holographic Dark Energy Cosmological Models in f(G) Theory", *New Astron.*, Vol. **80**, p. **101420**, Oct. **2020**, doi: 10.1016/j.newast.2020.101420.

[38] D. D. Pawar, R. V. Mapari, and P. K. Agrawal, "A modified holographic Ricci dark energy model in f (R, T) theory of gravity", *Journal of Astrophysics and Astronomy*, Vol. **40**, pp. **1-8**, **2019**.

[39] J. F. Zhang, Y. H. Li, and X. Zhang, "A global fit study on the new agegraphic dark energy model", *The European Physical Journal C*, Vol. **73**, No. **1**, p. **2280**, **2013**.

[40] A. Pourbagher, and A. Amani, "Thermodynamics of the viscous f (T, B) gravity in the new agegraphic dark energy model", *Modern Physics Letters A*, Vol. **35**, No. **20,** p. **2050166**, **2020**.

[41] B. Pourhassan, "Viscous modified cosmic Chaplygin gas cosmology", *International Journal of Modern Physics D*, Vol. **22**, No. **09**, p. **1350061**, **2013**.
[42] B. Pourhassan, and E. O. Kahya, "Extended Chaplygin gas model", *Results in Physics*, Vol. **4**, p. **101**, **2014**.
[43] E. O. Kahya, and B. Pourhassan, "The universe dominated by the extended Chaplygin gas", *Modern Physics Letters A*, Vol. **30**, No. **13**, p. **1550070**, **2015**.
[44] J. P. Campos, J. C. Fabris, R. Perez, O. F. Piattella, and H. Velten, "Does Chaplygin gas have salvation?", *The European Physical Journal C*, Vol. **73**, pp. **1-15**, **2013**.
[45] H. A. Buchdahl, "Non-linear Lagrangians and cosmological theory", *Monthly Notices of the Royal Astronomical Society*, Vol. **150**, No. **1**, pp. **1-8**, **1970**.
[46] K. Pawar, A. K. Dabre, N. T. Katre, "Anisotropic String Cosmological Model for Perfect Fluid Distribution in *f* (*T*) Gravity", vol. **10**, Issue **6**, pp. **1–7**, **2022**, doi: https://doi.org/10.26438/ijsrpas/v10i6.17.
[47] K. Pawar, N.T. Katre, A.K. Dabre, "Accelerating Expansion of the Universe with Dark Matter and Holographic Dark Energy in *f* (*T*) Gravity," *International Journal of Scientific Research in Physics and Applied Sciences*, Vol.**11**, Issue **2**, pp.**1-9**, **2023.**
[48] T. Harko, F. S. N. Lobo, S. Nojiri, and S. D. Odintsov, "f (R, T) gravity", *Physical Review D*, Vol. **84**, No. **2**, p. **024020**, **2011**.
[49] M. Sharif, and M. Zubair, "Anisotropic universe models with perfect fluid and scalar field in f (R, T) gravity", *Journal of the Physical Society of Japan*, Vol. **81**, No. **11**, p. **114005**, **2012**.
[50] M. Sharif, and M. Zubair, "Study of Bianchi I anisotropic model in f (R, T) gravity", *Astrophysics and Space Science*, Vol. **349**, pp. **457-465**, **2014**.
[51] D. R. K. Reddy, R. L. Naidu, and B. Satyanarayana, "Kaluza-Klein Cosmological Model in f (R, T) Gravity", *International Journal of Theoretical Physics*, Vol. **51**, pp. **3222-3227**, **2012**.
[52] D. R. K. Reddy, R. Santikumar, and R. L. Naidu, "Bianchi Type III Cosmological Models in f(R, T) Theory of Gravity", Astrophysics and Space Science, Vol. **342**, pp. **249-252**, **2012**.
[53] M. S. Berman, "A special law of variation for Hubble's parameter", *Nuovo Cimento B Serie*, Vol. **74**, pp. **182-186**, **1983**.
[54] K. S. Adhav, "LRS Bianchi type-I cosmological model in f (R, T) theory of gravity", *Astrophysics and space science*, Vol. **339**, pp. **365-369**, **2012**.
[55] S. Chandel, and S. Ram, "Anisotropic Bianchi type-III perfect fluid cosmological models in f (R, T) theory of gravity", *Indian Journal of Physics*, Vol. **87**, pp. **1283-1287, 2013**.
[56] G. C. Samanta, "Kantowski-Sachs universe filled with perfect fluid in f (R, T) theory of gravity", *International Journal of Theoretical Physics*, Vol. **52**, pp. **2647-2656**, **2013**.
[57] V. U. M. Rao, and D. Neelima, "Perfect-fluid Einstein-Rosen universe in f (R, T) gravity", *The European Physical Journal Plus*, Vol. **128**, pp. **1-8**, **2013**.
[58] V. U. M. Rao, and D. Neelima, "Bianchi type-VI_0 perfect fluid cosmological model in a modified theory of gravity", *Astrophysics and Space Science*, Vol. **345**, pp. **427-430**, **2013**.
[59] G. C. Samanta, "Universe filled with dark energy (DE) from a wet dark fluid (WDF) in f (R, T) gravity", *International Journal of Theoretical Physics*, Vol. **52**, pp. **2303-2315**, **2013**.
[60] V. U. M. Rao, K. V. S. Sireesha, and D. C. Papa Rao, "Perfect fluid cosmological models in a modified theory of gravity", *The European Physical Journal Plus*, Vol. **129**, pp. **1-8**, **2014**.
[61] A. K. Yadav, "Bianchi-V string cosmology with power law expansion in f (R, T) gravity", *The European Physical Journal Plus*, Vol. **129**, No. **9**, p. **194**, **2014**.
[62] B. Mishra, and P. K. Sahoo, "Bianchi type VI h perfect fluid cosmological model in f (R, T) theory", *Astrophysics and Space Science*, Vol. **352**, pp. **331-336**, **2014**.
[63] P. K. Sahoo, and B. Mishra, "Kaluza–Klein dark energy model in the form of wet dark fluid in f (R, T) gravity", *Canadian Journal of Physics*, Vol. **92**, No. **9**, **2014**.
[64] P. K. Sahoo, B. Mishra, and G. C. Reddy, "Axially symmetric cosmological model in f (R, T) gravity", *The European physical journal plus*, Vol. **129**, pp. **1-8**, **2014**.
[65] N. Ahmed, and A. Pradhan, "Bianchi type-V cosmology in f (R, T) gravity with Λ (T)", *International Journal of Theoretical Physics*, Vol. **53**, No. **1**, pp. **289-306**, **2014**.
[66] V. U. M. Rao, and D. C. Papa Rao, "Five dimensional anisotropic dark energy model in f (R, T) gravity", *Astrophysics and Space Science*, Vol. **357**, pp. **1-5**, **2015**.
[67] P. K. Sahoo, and M. Sivakumar, "LRS Bianchi type-I cosmological model in f (R, T) theory of gravity with Λ (T)", *Astrophysics and Space Science*, Vol. **357**, pp. **1-12**, **2015**.
[68] S. D. Katore, V. R. Chirde, and S. P. Hatkar, "Domain walls in f (R, T) theory of gravitation", *International Journal of Theoretical Physics*, Vol. **54**, pp. **3654-3664**, **2015**.
[69] P. K. Sahoo, S. K. Sahu, and A. Nath, "Anisotropic Bianchi-III cosmological model in f (R, T) gravity", *The European Physical Journal Plus*, Vol. **131**, No. **1**, p. **18**, **2016**.
[70] G. P. Singh, B. K. Bishi, and P. K. Sahoo, "Cosmological constant Λ in f (R, T) modified gravity", *International Journal of Geometric Methods in Modern Physics*, Vol. **13**, No. **05**, p. **1650058**, **2016**.
[71] P. H. R. S. Moraes, "Cosmology from f (R, T) theory in a variant speed of light scenario." *International Journal of Theoretical Physics*, Vol. **55**, pp. **1307-1314**, **2016**.
[72] D. Sofuoğlu, "Rotating and expanding Bianchi type-IX model in f (R, T) theory of gravity", *Astrophysics and Space Science*, Vol. **361**, pp. **1-7**, **2016**.
[73] P. Sahoo, "LRS Bianchi type-I string cosmological model in gravity", *Fortschritte der Physik*, Vol. **64**, No. **4-5**, pp. **414-415**, **2016**.
[74] P. K. Sahoo, B. Mishra, P. Sahoo, and S. K. J. Pacif, "Bianchi type string cosmological models in f (R, T) gravity", *The European Physical Journal Plus*, Vol. **131**, pp. **1-12**, **2016**.
[75] G. C. Samanta, and R. Myrzakulov, "Imperfect fluid cosmological model in modified gravity", *Chinese Journal of Physics*, Vol. **55**, No. **3**, pp. **1044-1054**, **2017**.
[76] P. K. Agrawal, and D. D. Pawar, "Plane symmetric cosmological model with quark and strange quark matter in f (R, T) theory of gravity", *Journal of Astrophysics and Astronomy*, Vol. **38**, pp. **1-7**, **2017**.
[77] P. K. Agrawal, and D. D. Pawar, "Magnetized domain wall in f (R, T) theory of gravity", *New Astronomy*, Vol. **54**, pp. **56-60**, **2017**.
[78] R. Zia, D. C. Maurya, and A. Pradhan, "Transit dark energy string cosmological models with perfect fluid in F (R, T)-gravity", *International Journal of Geometric Methods in Modern Physics*, Vol. **15**, No. **10**, p. **1850168**, **2018**.
[79] Y. Aditya, and D. R. K. Reddy, "Dynamics of perfect fluid cosmological model in the presence of massive scalar field in f (R, T) gravity", *Astrophysics and Space Science*, Vol. **364**, No. **1**, p. **3**, **2019**.
[80] D. D. Pawar, R. V. Mapari, and P. K. Agrawal, "A modified holographic Ricci dark energy model in f (R, T) theory of gravity", *Journal of Astrophysics and Astronomy*, Vol. **40**, pp. **1-8**, **2019**.
[81] D. D. Pawar, R. V. Mapari, and J. L. Pawade, "Perfect fluid and heat flow in f (R, T) theory", *Pramana*, Vol. **95**, No. **1**, p. **10**, **2021**.
[82] B. P. Brahma, and M. Dewri, "Bulk Viscous Bianchi Type-V Cosmological Model in f (R, T) Theory of Gravity", *Frontiers in Astronomy and Space Sciences*, Vol. **9**, p. **11**, **2022**.
[83] C. B. Collins, E. N. Glass, and D. A. Wilkinson, "Exact spatially homogeneous cosmologies" *General Relativity and Gravitation*, Vol. **12**, pp. **805-823**, **1980**.
[84] C. B. Collins, and S. W. Hawking, "Why is the universe isotropic?", *Astrophysical* Journal, Vol. **180**, pp. **317-334**, **1973**.
[85] B. Huang, S. Li, and Y. Ma, "Five-dimensional metric f (R) gravity and the accelerated universe", *Physical Review D*, Vol. **81**, No. **6**, p. **064003**, **2010**.
[86] V. U. M. Rao, D. Neelima, and K. V. S. Sireesha, "Cosmological Models in a Modified Theory of Gravity", *Prespacetime Journal*, Vol. **4**, No. **3**, **2013**.
[87] C. E. Cunha, M. Lima, H. Oyaizu, J. Frieman, and H. Lin, "Estimating the redshift distribution of photometric galaxy samples– II. Applications and tests of a new method", *Monthly Notices of the Royal Astronomical Society*, Vol. **396**, No. **4**, pp. **2379-2398**, **2009**.
[88] F. Rajabi, and K. Nozari, "Energy condition in unimodular f (R, T) gravity", *The European Physical Journal C*, Vol. **81**, pp. **1-13**, **2021**.

AUTHORS PROFILE

Dr. Kalpana N. Pawar pursued M.Sc. and Ph.D. in Mathematics from the Institute of Science, Nagpur, in 1997 & 2003 respectively. She started her teaching career as an Assistant Professor at Institute of Science, Nagpur, and currently working as Professor & Head, Department of Mathematics, Shri R. R. Lahoti Science College, Morshi since 2013. She is a Ph.D. supervisor of RTM Nagpur University, Nagpur since 2008 and SGB Amravati University, Amravati since 2019. She had guided five research scholars for award of M. Phil and Ph. D. and five research scholars are prently working under her supervision for the award of Ph.D. She had published various research papers in reputed national and international journals. Her main research area is General Theory of Relativity and Cosmology, Alternating Theories of Gravitation, Modified Theories of Gravitation, and Fuzzy Controllers. She had in all 25 years of teaching experience and 24 years of research experience.

Mr. N.T. Katre (M.Sc., B.Ed., SET) is currently pursuing a Ph.D. and working as an Assistant Professor and Head, Department of Mathematics, Nabira Mahavidyalaya, Katol, Dist. Nagpur (M.S.), India. He has 23 years of teaching experience at U.G., 12 years of teaching experience at P.G., and 7 years of Research Experience. He is a life member of the Indian Mathematical Society since 2007. His main research work focuses on Gravitation Theories/ General Relativity/ Cosmology. He has published more than 17 Books and 12 research papers in reputed national/international journals.

