International Journal of Scientific Research in Physics and Applied Sciences
Vol.**12**, Issue.**5**, pp.**01-06**, October **2024**
E-ISSN: 2348-3423
Available online at: www.isroset.org

Research Article

Bianchi Type-VI_O String Cosmological Model in *f(R,T)* Theory of Gravity

S. P. Gaikwad[1*], A. K. Dabre[2], K. R. Mule[3]

[1]Dept. of Mathematics, L. K. D. K. Banmeru Science College, Lonar, India
[2]Dept. of Mathematics, Shri R. R. Rahoti Science College, Morshi, India
[3]Dept. of Mathematics, S. D. M. B. Science and Arts College, Shegaon, (M.S.), India

**Corresponding Author: gaikwad1.618@gmail.com*

Received: 07/Aug/ 2024; **Accepted**: 09/Sept/2024; **Published**: 31/Oct/2024

Abstract— This paper investigates a spatially homogeneous and anisotropic Bianchi type-VIo universe with a string of clouds in the framework of $f(R,T)$ gravitational theory. The field equations were found by considering the proportionality of the expansion scalar θ and the shear scalar σ. Some cosmologically crucial physical and kinematical characteristics of the model are determined, and their geometric relevance is addressed.

Keywords— Bianchi type-VI_o, Cosmic string, $f(R,T)$ gravity.

1. Introduction

The universe appears to be expanding at an accelerating rate, according to recent observations of high redshift supernovae type-Ia [1-4], although the FRW model indicates the opposite. This acceleration can be explained by two alternative methods. The first suggests that dark energy is the cause of this acceleration. An alternative strategy is to modify the gravitational theory. Numerous modified gravity theories have recently been devised to explain the existence of dark energy and dark matter, as well as the process driving the universe's late-time acceleration.

Harko et al. [5] established a modified gravitational theory called as $f(R,T)$ gravity. Many physicists have been interested in this concept because it appears to serve as a gravitational equivalent to dark energy. In $f(R,T)$ gravitational theory, Adhav [6], Sharif and Zubair [7], and Mahanta [8] investigated the Bianchi type-I cosmic model. Naidu et al. [9], Ahmed & Pradhan [10], and Pawar & Dabre [11] explored the Bianchi type-V model within the context of $f(R,T)$ gravity. In $f(R,T)$ gravity, Shaikh and Bhoyar [12] explored plane symmetric universes. Based on the prior investigations, $f(R,T)$ theory appears to be a more feasible explanation for the universe's accelerating phase.

Researcher’s curiosity about anisotropic cosmological models has developed in response to anomalies discovered in large-scale structural investigations and the cosmic microwave background (CMB). The significance of Bianchi models lies in their homogeneity and anisotropy, which allow for the study of the universe's isotropization process across time. According to Barrow [13], Bianchi type- VI_0 cosmological models are isotropic in certain situations and provide greater clarity for some cosmic challenges, like primordial helium abundance. Krori et al. [14] studied massive string cosmic models for Bianchi type- VI_0 metric. Several exact solutions for Bianchi type- VI_0 model regardless of a magnetic field have been studied by Tikekar and Patel [15]. Bali et al. have explored LRS Bianchi type- VI_0 metric with a unique free gravitational field [16]. Given the growing interest among cosmologists, we propose examining cosmic evolution within the boundaries of Bianchi type- VI_0 space-time.

By characterizing the initial stages of the universe in the form of vibrating strings rather than particles, the concept of strings integrates all matter and forces together in a unified conceptual paradigm. There seems to be an increase of curiosity in studying string fluid cosmological models since they combine all of the basic physics principles and the development of the initial universe. Cosmological string-fluids could provide a promising explanation for a variety of cosmological occurrences in the study of the universe's evolution.

In the framework of general relativity, Mahanta and Mukharjee [17], Bhattacharjee and Baruah [18], investigated numerous features of cosmic strings. Letelier [19] developed a universe model in general relativity utilizing string-fluid. Maurya et al. [20] examined transit string dark energy models in $f(R,T)$ gravity. Zia et al. [21] investigated a modified $f(R,T)$ gravity model with a variable equation of state based on strings. The acceleration of the Bianchi type-III perfect fluid string model in $f(R,T)$ gravity context was investigated by [22] Rani et al. By using the hybrid expansion law, Ram and Chandel [23] investigated the behavior of a magnetized string model. Rani et al. [22] examined the acceleration of the Bianchi type-III perfect fluid string cosmological model within the framework of $f(R,T)$ gravity. Assuming the hybrid expansion law, Ram and Chandel [23] investigated the behavior of a magnetized string universe. A large string magnetized barotropic perfect fluid cosmic model in general relativity was studied by Bali et al. [24] using the Bianchi type-I metric. In the $f(T)$ gravity domain, Pawar et al. [25] investigated perfect fluid string cosmic models and found consistent results to the field equations. Pradhan et al. [26] studied the string of clouds in the context of perfect fluid and declining vacuum energy density. The inhomogeneous Bianchi type-I cosmological model for stiff perfect fluid distribution was studied by Tripathi et al. [27]. Again, Pawar et al. [28,29] vigorously investigated string cosmological models coupled with zero mass scalar field as well as bulk viscous fluid in the context of *f(T)* & *f(R)* gravities.

Influenced by the foregoing discussion, we examined a spatially homogeneous as well as anisotropic Bianchi type-VI_o space-time with cosmic string in reference to $f(R,T)$ gravity theory. This research primary goal is to investigate this Bianchi type-VI_o model in $f(R,T)$ modified gravity, considering the different concerns about the late time cosmic acceleration in the cosmic string presence. The following is the paper's outline: Section 2 presents the methods and a quick overview of $f(R,T)$ gravity. We have derived field equations in section 3. Section 4 included some physical and kinematical parameters in the physical interest of cosmology as well as field equation solutions. Finally, we provided a summary of the findings in section 5.

2. Methodology and Brief Review of *f(R,T)* Gravity

The $f(R,T)$ gravity theory is a modification or extension of Einstein's General Theory of Relativity presented by Harko et al. [5]. This theory's field equations have been derived from the Hilbert variational principle of Einstein and are provided by

$$S=\frac{1}{2k}\int f(R,T)\sqrt{-g}\;d^4x+\int L_m\sqrt{-g}\;d^4x, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R, the trace T of the stress-energy tensor of the matter T_{ij} and L_m is the matter Lagrangian density. For matter, the stress-energy tensor T_{ij} is defined as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,L_m)}{\delta g^{ij}}, \tag{2}$$

and its trace given by $T_{ij}=g^{ij}T_{ij}$. Assuming that the Lagrangian density of matter L_m is only dependent on the components of the metric tensor g_{ij} instead of its derivative, (2) results in

$$T_{ij}=g_{ij}L_m-2\frac{\partial(L_m)}{\partial g^{ij}}. \tag{3}$$

The field equations of $f(R,T)$ gravity theory are produced by varying the action S in (1) with regard to the metric tensor components g^{ij} is given by

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+f_R(R,T)(g_{\mu\nu}\nabla^i\nabla_j-\nabla_i\nabla_j)=8\pi T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\Theta_{ij} \tag{4}$$

with $\Theta_{ij}=g^{lm}\left(\frac{\delta T_{lm}}{\delta g^{ij}}\right)$,

which follows from the relation $\delta\left(\frac{g^{lm}T_{lm}}{\delta g^{ij}}\right)=T_{ij}+\Theta_{ij}$, and here $f_R(R,T)=\frac{\partial f(R,T)}{\partial R}$, $f_T(R,T)=\frac{\partial f(R,T)}{\partial T}$, and ∇_i are the covariant derivatives.

The contraction of (4) yields

$$f_R(R,T)R+3\nabla^i\nabla_j f_R(R,T)-2f(R,T)=(8\pi-f_T(R,T))T-f_T(R,T)\Theta_{ij}, \tag{5}$$

with $\Theta=g^{ij}\Theta_{ij}$. Eliminating $\nabla^i\nabla_j f_R(R,T)$ from (4) and (5) we get,

$$f_R(R,T)\left(R_{ij}-\frac{1}{3}Rg_{ij}\right)+\frac{1}{6}f(R,T)g_{ij}=F_1+F_2, \tag{6}$$

where, $F_1=\left(8\pi-f_T(R,T)\right)\left(T_{ij}-\frac{1}{3}T\,g_{ij}\right)$,

$$F_2=-f_T(R,T)\Theta_{ij}+\frac{1}{3}f_T(R,T)\Theta g_{ij}+\nabla_i\nabla_j f_R(R,T).$$

From (2) we have

$$\frac{\delta T_{ij}}{\delta g^{lm}}=\left(\frac{\delta g_{ij}}{\delta g^{lm}}+\frac{1}{2}g_{ij}g_{lm}\right)L_m-2\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}-\frac{1}{2}g_{ij}T_{lm}. \tag{7}$$

Using relation $\frac{\delta_{ij}}{\delta g^{lm}}=-g_{i\gamma}g_{j\sigma}\delta_{lm}^{\gamma\sigma}$ with $\delta_{lm}^{\gamma\sigma}=\frac{\delta T^{\gamma\sigma}}{\delta g^{lm}}$, Θ_{ij} is obtained as

$$\Theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}. \tag{8}$$

We assume that stress energy tensor of matter is given by

$$T_{ij}=(\rho+p)u_i u_j-pg_{ij}, \tag{9}$$

and the matter Lagrangian can be taken as $L_M=-p$ and we have $u^iu_i=1$, and $u^i\nabla_j u_i=0$. Using (8) we have obtained the expression for the variation of stress energy as

$$\Theta_{ij}=-2T_{ij}-pg_{ij}. \tag{10}$$

The cosmological implications of the model put forth by Harko et al., [5] are examined in this work by taking

$$f(R,T) = R + 2f(T), \quad (11)$$

where $f(T)$ is an arbitrary function of the trace of the stress-energy tensor of matter.

Combining (10) and (11) the field equation of $f(R,T)$ gravity from (4) leads to

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + \left[2pf'(T) + f(T)\right]g_{ij}. \quad (12)$$

here the differentiation with regard to cosmic time t is indicated by the overhead prime.

4. Metric and Field Equations

We consider Bianchi type-VI_0 metric as

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2x}dy^2 - C^2e^{-2x}dz^2, \quad (13)$$

where, the scale factors A, B and C are functions of cosmic time t only.

We consider the energy momentum tensor for cosmic strings as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \quad (14)$$

where ρ is the energy density of the string cloud and λ is the string tension density. Here we consider the string cloud source along the x-axis, which is the axis of symmetry, an ortho-normalization of u_i and x_i is the given by

$$g_{ij}u_iu_j = -x_ix_j = 1 \text{ and } u_ix_i = 0 \quad (15)$$

$$\rho = \rho_p + \lambda. \quad (16)$$

where ρ_p being the rest energy of particles attached to the strings. As pointed out by Letelier [19], λ may be positive or negative.

The energy condition leads to $\rho \geq 0$ and $\rho_p \geq 0$, leaving the sign of λ unrestricted.

By choosing the function given by Harko et al., [5] as

$$f(T) = \mu T, \quad (17)$$

where μ is a constant.

Now assuming the co-moving coordinate system, we obtained the field equations for Bianchi type-VI_0 space-time (13), from (12) in the framework of $f(R,T)$ gravity as

$$-\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = (2p + \rho + 3\lambda)\mu + 8\pi\lambda, \quad (18)$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} + \frac{1}{A^2} = 2p\mu + (\lambda + \rho)\mu, \quad (19)$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} + \frac{1}{A^2} = 2p\mu + (\lambda + \rho)\mu \quad (20)$$

$$-\frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} + \frac{1}{A^2} = (2p + 3\rho + \lambda)\mu + 8\pi\rho, \quad (21)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \quad (22)$$

where the overhead dot (.) denotes the derivative with respect to cosmic time t. Here we have five highly non-linear differential field equations with six unknowns namely: A, B, C, p, λ and ρ.

5. Solutions of Field Equations

Now, we take into account the average scale factor a and spatial volume V as

$$a = \sqrt[3]{ABC}, \quad V = a^3. \quad (23)$$

From (22) we get

$$B = \kappa C, \quad (24)$$

where κ is an integrating constant but without loss of generality we consider $\kappa = 1$.

Now, using (24) and subtracting (19) from (18) we get

$$(8\pi + 2\mu)\lambda = \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{2}{A^2} - \frac{\dot{B}^2}{B^2} \quad (25)$$

Similarly, using (24) and subtracting (21) from (20) we get

$$(8\pi + 2\mu)\rho = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2}. \quad (26)$$

For the deterministic solutions, we consider the expansion scalar Θ is proportional to the shear scalar σ which lead to the analytic relation

$$B = A^n. \quad (27)$$

Now we consider hybrid scale factor

$$a = t^{\alpha}e^{\beta t}, \quad (28)$$

where α and β are positive constants.

From (23), (24), (27) & (28) we get

$$A(t) = \left(t^{\alpha}e^{\beta t}\right)^{\frac{3n}{(n+2)}}, \quad B(t) = C(t) = \left(t^{\alpha}e^{\beta t}\right)^{\frac{3}{(n+2)}}. \quad (29)$$

Thus metric (13) becomes

$$ds^2 = dt^2 - \left(t^{\alpha}e^{\beta t}\right)^{\frac{6n}{(n+2)}}dx^2 - \left(t^{\alpha}e^{\beta t}\right)^{\frac{6}{(n+2)}}e^{2x}dy^2 - \left(t^{\alpha}e^{\beta t}\right)^{\frac{6}{(n+2)}}e^{-2x}dz^2. \quad (30)$$

Now, we find some kinematical space-time quantities of physical interest in cosmology, defined and obtained as

Spatial volume is defined and obtained as

$$V = a^3(t) = t^{3\alpha}e^{3\beta t} \quad (31)$$

The mean Hubble parameter, which expresses the volumetric expansion rate of the universe, is

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{\beta t + \alpha}{t}. \quad (32)$$

The expression for the expansion scalar θ, which deals with the expansion of the universe, is obtained as

$$\theta = 3H = \frac{3(\beta t + \alpha)}{t}. \tag{33}$$

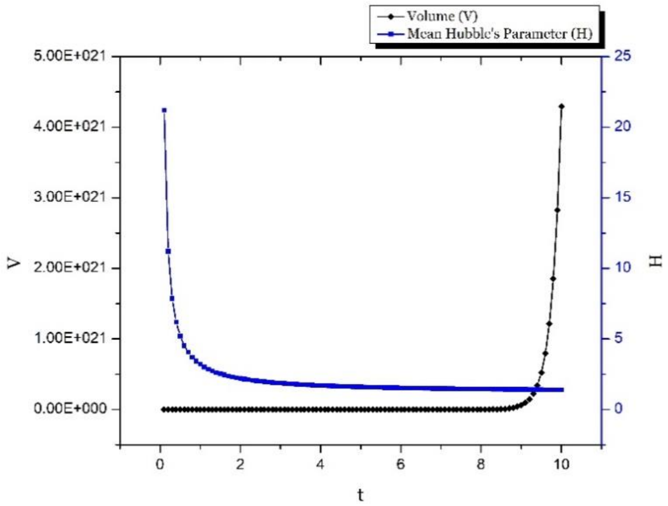

Figure 1. Behaviour of Volume & Hubble's Parameter Vs. time for $\alpha = 2,\ \beta = 1.2$

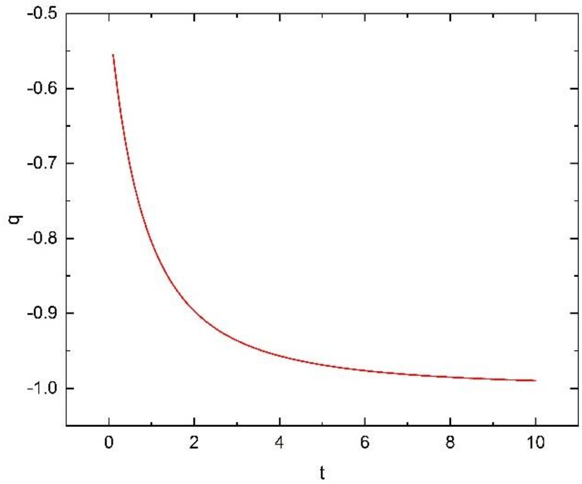

Figure 2. Behaviour of Deceleration Parameter Vs. time for $\alpha = 2,\ \beta = 1.2$

The rate of expansion of the universe is evaluated by anisotropy parameter, which is given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \frac{2(n-1)^2}{(n+2)^2}. \tag{34}$$

Shear scalar is

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right]^2 = \frac{3(\beta t + \alpha)^2 (n-1)^2}{t^2 (n+2)^2}. \tag{35}$$

The deceleration parameter q is known to be a measure of cosmic acceleration is obtained as

$$q = -\frac{(\beta t + \alpha)^2 - \alpha}{(\beta t + \alpha)^2} = -1 + \frac{\alpha}{(\beta t + \alpha)^2}. \tag{36}$$

According to the results above, the volume increases exponentially (Figure 1) and does not have an initial singularity. Figure 1 also illustrates that Hubble's parameter is a decreasing function of cosmic time indicating that the model in equation (30) is expanding in nature, with a significant initial expansion rate that decreases with time. The behaviour of deceleration parameter have been addressed in Figure 2 from which it is been observed that the deceleration parameter decreases monotonically with increase in cosmic time, eventually approaching -1 when $t \to \infty$, indicating an accelerated cosmic evolution that is consistent with recent observational findings. From expression (34), the mean anisotropy parameter is constant throughout the expansion and hence the model does not approaches isotropy throughout the whole cosmic evolution. Furthermore, expression (35) states that the model holds shear throughout the evolution.

From (26), we have obtained the energy density as

$$\rho = \frac{3\left\{3\left[(\beta t + \alpha)^2 - \alpha\right]n^2 - 3\left[(\beta t + \alpha)^2 + \alpha\right]n - 2\alpha\right\}}{t^2 (n+2)^2 (8\pi + 2\mu)}. \tag{37}$$

From (25), we have obtained the tension density as

$$\lambda = \frac{3(n-1)\left[3(\beta t + \alpha)^2 - \alpha\right]\left(t^{\alpha} e^{\beta t}\right)^{\frac{6n}{(n+2)}} - 2(n+1)t^2}{t^2 (n+2)\left(t^{\alpha} e^{\beta t}\right)^{\frac{6n}{(n+2)}} (8\pi + 2\mu)}. \tag{38}$$

From (16), (37) & (38), the particle density is obtained as

$$\rho_p = \frac{\left(t^{\alpha} e^{\beta t}\right)^{-\frac{6n}{(n+2)}} (n+2)^2 t^2 - 9t^2\beta^2(n-1) - 18\beta\alpha(n-1)t + \left(-9\alpha^2 - 3\alpha\right)n + 9\alpha^2 - 6\alpha}{t^2 (n+2)^2 (4\pi + \mu)}. \tag{39}$$

Figure 3 represents the behaviour of energy, tension & particle density with increasing cosmic time. It is been observed that the energy density immediately drops at first and then gradually slows down over time. Furthermore, the physical behavior of the tension density demonstrates that it rises sharply within a range (t ∈ [0, 1]) from negative to positive, then gradually decreases and becomes constant (closed to zero). The particle density, on the other hand, rigorously decreases in a range (t ∈ [0, 1]), gradually increases subsequently, and eventually remains constant (near to zero). In the beginning when $\lambda < 0$ and $\rho_p > 0$ the universe is said to have particle-dominated phase but later on, not only is the emergence of strings seen, but the universe is also observed to be string-dominated. These observations are supported with [30].

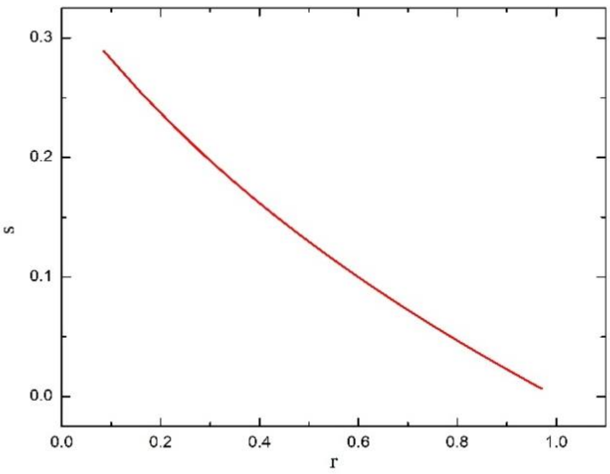

Figure 3. Behaviour of Energy, Tension & Particle Density Vs. time for $\alpha = 2, \beta = 1.2, \mu = 1.5, n = 3$

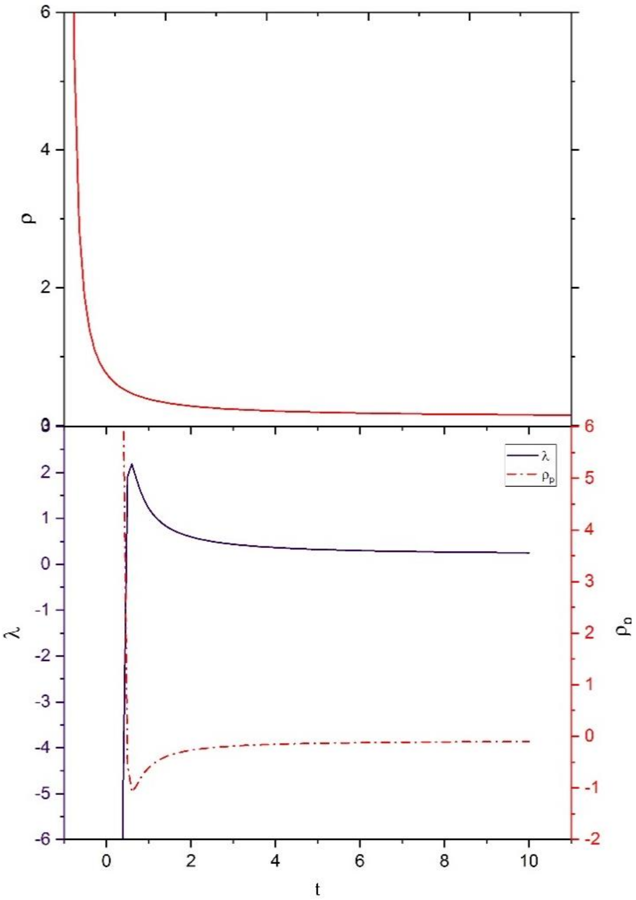

Figure 4. Behaviour of Statefinder Parameter for $\alpha = 2, \beta = 1.2, \mu = 1.5, n = 3$

Diagnostic Statefinder Parameters:

The statefinder parameters $\{r, s\}$ proposed by Sahni *et al.* [27] are useful in identifying different forms of dark energy. The primary goal of statefinder parameters is to differentiate between several DE models that are put forth from time to time in modern cosmology. For different DE models, the different sets of values of the pair are mentioned below:

- For ΛCDM model: (r = 1, s = 0),
- For SCDM model: (r = 1, s = 1),
- For HDE model: (r = 1, s = 2/3),
- For CG model: (r > 1, s < 0),
- For Quintessence model: (r < 1, s > 0).

The pair of statefinder parameters $\{r, s\}$ are obtained as follows:

$$r = \frac{\dddot{a}}{aH^3} = \frac{\beta\left(3\alpha^2 + (3\beta t - 3)\alpha + \beta^2 t^2\right)t + \left((\alpha - 3)\alpha + 2\right)\alpha}{(\beta t + \alpha)^3}, \qquad (40)$$

$$s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} = \frac{2}{3}\frac{\alpha(3\beta t + 3\alpha - 2)}{\left(3(\beta t + \alpha)^2 - 2\alpha\right)(\beta t + \alpha)}. \qquad (41)$$

Fig. 4 depicts the variation of statefinder parameter s versus r for appropriate choice of constants. It is observed that the parameter s > 0 and the parameter r < 1 throughout the evolution of the universe. Hence, from the discussed observations it is been confirmed that the model so derived here is the Quintessence model.

6. Conclusion

In this study, we have presented the Bianchi type-VI_0 space-time with cosmic string in the context of $f(R,T)$ gravity. The derived model lacks the existence of an initial singularity and exhibits the behavior consistent with present universe, which is expanding and accelerating at the same time, and is in line with recent observations. Furthermore, the anisotropic nature of the model is noted. As cosmic time increases, the energy density decreases. The tension density rises sharply within a range (t ∈ [0, 1]) and then gradually decreases to attain constant. However, the particle density strictly drops within a range (t ∈ [0, 1]), increases gradually thereafter, and eventually remains constant (closed to zero) in negative. Furthermore, it is observed that, in the beginning when $\lambda < 0$ and $\rho_p > 0$ the universe is said to have particle-dominated phase but later on, not only is the emergence of strings seen, but the universe is also observed to be string-dominated. These observations are supported with [29,30].

Conflict of Interest
Authors declare that they do not have any conflict of interest.

Funding Source
No funding received

Authors' Contributions
Author-1 reviewed literature and identified the problem of study and wrote the first draft of the manuscript. Author-2 involved in solving equation and plotting graph using software. Author-3 reviewed and edited the manuscript and approved the final version of the manuscript.

References

[1] S. Perlmutter et al., 'Measurements* of the Cosmological Parameters Ω and Λ from the First Seven Supernovae at z⩾ 0.35', *The astrophysical journal*, Vol.**483**, No.**2**, pp.**565, 1997.**

[2] S. Perlmutter et al., 'Discovery of a supernova explosion at half the age of the Universe', *Nature*, Vol.**391**, No.**6662**, pp.**51–54, 1998**. https://doi.org/10.1038/34124

[3] A. G. Riess et al., 'Observational evidence from supernovae for an accelerating universe and a cosmological constant', *The astronomical journal*, Vol.**116**, No.**3**, pp.**1009, 1998.**

[4] C. L. Bennett et al., 'The microwave anisotropy probe* mission', *The Astrophysical Journal*, Vol.**583**, No.1, pp.1, **2003.**

[5] T. Harko, F. S. N. Lobo, S. Nojiri, and S. D. Odintsov, 'f (R, T) gravity', *Physical Review D—Particles, Fields, Gravitation, and Cosmology*, Vol.**84**, No.**2**, p.**024020, 2011.** https://doi.org/10.1103/PhysRevD.84.024020

[6] M. V. Dawande, K. S. Adhav, and S. S. Nerkar, 'LRS Bianchi Type-I Universe in F (T) Theory of Gravity', *Advances in Astrophysics*, Vol.**2**, No.**2**, pp.**117–125, 2017.**

[7] M. Sharif, M. Zubair, J. Phys. Soc. Jpn. 81, 114005, 2012.

[8] K. L. Mahanta, 'Bulk viscous cosmological models in f(R, T) theory of gravity', *Astrophysics and Space Science*, vol.**353**, pp.**683–689, 2014.** https://doi.org/10.1007/s10509-014-2040-6

[9] R. L. Naidu, D. R. K. Reddy, T. Ramprasad, and K. V. Ramana, 'Bianchi type-V bulk viscous string cosmological model in f (R, T) gravity', *Astrophysics and Space Science*, Vol.**348**, pp.**247–252, 2013**. https://doi.org/10.1007/s10509-013-1540-0

[10] N. Ahmed and A. Pradhan, 'Bianchi type-V cosmology in f (R, T) gravity with Λ (T)', *International Journal of Theoretical Physics*, Vol.**53**, No.**1**, pp.**289–306, 2014**. https://doi.org/10.1007/s10773-013-1809-7

[11] K. Pawar and A. Dabre, 'Perfect Fluid Coupled String Universe in f (R, T) Gravity', *Journal of Scientific Research*, Vol.**15**, No.**3**, pp.**695–704, 2023**. http://doi.org/10.3329/jsr.v15i3.64173

[12] A. Y. Shaikh and S. R. Bhoyar, 'Plane Symmetric Universe with Λ in f (R, T) Gravity', *Prespacetime Journal*, Vol.**6**, No.**11**, **2015.**

[13] J. D. Barrow, 'Helium formation in cosmologies with anisotropic curvature', Monthly Notices of the Royal *Astronomical Society*, Vol.**211**, No.2, pp.**221–227, 1984.** https://doi.org/10.1093/mnras/211.2.221

[14] K. D. Krori, T. M. Chaudhuri, and C. R. Mahanta, 'C R. and Mazumdar', *A. Gen. Relativ. Gravit*, Vol.**22**, No.**990**, pp.**123, 1990.**

[15] R. Tikekar and L. K. Patel, 'Some exact solutions in Bianchi VI_0 string cosmology', *Pramana,* Vol.**42**, pp.**483–489, 1994.** https://doi.org/10.1007/BF02847130

[16] R. Bali, R. Banerjee, and S. K. Banerjee, 'Some LRS Bianchi Type VI_0 Cosmological Models with Special Free Gravitational Fields', *Electronic J. of Theoretical Phy.*, Vol.**6**, No.**21**, **2009.**

[17] P. Mahanta and A. Mukherjee, 'String models in Lyra geometry', *Indian Journal of Pure and Applied Mathematics*, Vol.**32**, No.**2**, pp.**199–204, 2001.**

[18] R. Bhattacharjee and K. K. Baruah, 'String cosmologies with a scalar field', *Indian J. Pure Appl. Math.* vol. 32, pp 7–53, 2001.

[19] P. S. Letelier, 'String cosmologies', *Physical review D*, vol. 28, no. 10, p. 2414, **1983**.

[20] D. C. Maurya, A. Dixit, and A. Pradhan, 'Transit string dark energy models in f (Q) gravity', *International Journal of Geometric Methods in Modern Physics*, vol. 20, no. 08, p. 2350134, **2023**. https://doi.org/10.1142/S0219887823501347

[21] R. Zia, D. C. Maurya, and A. Pradhan, 'Transit dark energy string cosmological models with perfect fluid in F (R, T)-gravity', *International Journal of Geometric Methods in Modern Physics*, vol. 15, no. 10, p. 1850168, **2018**. https://doi.org/10.1142/S0219887818501682

[22] S. Rani, J. K. Singh, and N. K. Sharma, 'Bianchi type-III magnetized string cosmological models for perfect fluid distribution in f (R, T) gravity', *International Journal of Theoretical Physics*, Vol.**54**, pp.**1698–1710**, **2015**. https://doi.org/10.1007/s10773-014-2371-7

[23] S. Ram and S. Chandel, 'Dynamics of magnetized string cosmological model in f (R, T) gravity theory', *Astrophysics and Space Science*, Vol.**355**, pp.**195–202**, **2015**. https://doi.org/10.1007/s10509-014-2160-z

[24] R. Bali, U. K. Pareek, and A. Pradhan, 'Bianchi type-I massive string magnetized barotropic perfect fluid cosmological model in general relativity', *Chinese Physics Letters*, vol. 24, no. 8, p. 2455, **2007**.

[25] K. Pawar, A. K. Dabre, and N. T. Katre, 'Anisotropic String Cosmological Model for Perfect Fluid Distribution in f (T) Gravity', *Int. J. Sci. Res. in Physics and Applied Sciences Vol*, vol. 10, no. 6, **2022**. https://doi.org/10.26438/ijsrpas/v10i6.17

[26] A. Pradhan, H. Amirhashchi, and H. Zainuddin, 'Exact solution of perfect fluid massive string cosmology in Bianchi type III space-time with decaying vacuum energy density Λ', *Astrophysics and Space Science,* vol. 331, no. 2, pp. 679–687, **2011**. https://doi.org/10.1007/s10509-010-0469-9

[27] A. Tyagi, S. Parikh, and Tripathi B. R. 'Inhomogeneous Bianchi Type-I Cosmological Model For Stiff Perfect Fluid Distribution', *Prespacetime Journal,* vol 7(12), **2016.**

[28] K. Pawar, A. K. Dabre, 'Bulk Viscous String Cosmological Model with Power Law Volumetric Expansion in Teleparallel Gravity', *Astrophysics*, Vol.**66,** Issue.1, pp.**114-126, 2023.**

[29] K. Pawar, A. K. Dabre, and P. Makode. 'Plane Symmetric String Cosmological Model with Zero Mass Scalar Field in f(R) Gravity', *Astrophysics,* Vol.**66**, No.3, pp.**353-365, 2023.**

[30] K. Pawar and A. K. Dabre, 'Bulk Viscous String Cosmological Model with Constant Deceleration Parameter in Teleparallel Gravity', *Int. J. Sci. Res. in Physics and Applied Sciences*, Vol.**10**, No.**6**, **2022**. DOI: https://doi.org/10.26438/ijsrpas/v10i6.816

AUTHORS PROFILE

Mr. S. P. Gaikwad pursued his M.Sc in Mathematics from Pune University, India. He is currently working as Asst. Professor and Head at Dept. of Mathematics, LKDK Banmeru Science College, Lonar, India. He is pursuing Ph.D in cosmology and modified gravity.

Dr. A. K. Dabre pursued his M.Sc in Mathematics from the Dept. of Mathematics, S. G. B. A. University, Amravati (M.S.), India. He has completed Ph.D in cosmology and modified gravity theories. He is currently working as Asst. Professor of Mathematics, at Shri R. R. Lahoti Science College, Morshi.

Dr. K. R. Mule pursued M.Sc. in Mathematics from Shri R. L. T. College of Science, Akola . He pursued M.Phil. from Periyar University, Salem and Ph.D from R.D.I.K & K.D Mahavidhlya Badnera Rly, Amravati (M.S.), India. He is currently working as Associate Professor & Head, Department of Mathematics, S. D. M. B. Science and Arts College, Shegaon, (M.S.), India. He is a Ph.D. supervisor for SGB Amravati University, Amravati. He had published around 22 research papers in reputed national and international journals and also actively participated in various national and international conferences.

