

Amplification of Plasma Wave through non-linear wave-particle interaction in ionospheric plasma

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Abstract— Wave energy up conversion of high frequency plasma wave in presence of low frequency plasma wave are studied in both space and laboratory plasma. In the ionosphere different forms of free energy sources are coexisting and energy exchange process takes place among plasma waves and particles. Through plasma maser effect ionospheric plasma particles may be transferred energy nonlinearly to high frequency mode through a modulated electric field in the presence of electron density gradients. In this paper we have explained how energy from drift wave is transferred to non-resonant ion sound wave. Including plasma inhomogeneity effect in our model the growth rate of ion sound wave in the Earth's ionosphere in presence of drift wave turbulence estimated using observational data.

Keywords—Ionospheric Plasma, Ion sound Wave, Drift Wave, Plasma Maser effect, Non-linear force

I. INTRODUCTION

The Earth's ionosphere is a cold, low β and weakly collisional plasma and can sustain plasma waves and instabilities. Perturbation develops in this open plasma system for the entry of energy and momentum fluxes through particles and waves and for some physical factors like solar wind, density gradients, magnetic field gradients and thermal gradients. Plasma instabilities are observed in the ionosphere mainly at the equatorial latitudes and in the auroral zone [1, 2, 3, 4, 5].

In this study, we investigate the amplification of electrostatic high frequency ion sound wave in the presence drift instability on the basis of a wave-particle interaction process called plasma maser effect. Plasma maser interaction is wave energy up conversion process. We consider two low frequency resonant plasma wave and high frequency nonresonant plasma wave are present in the system. The plasma particles which are in phase relation with a resonant mode may acquire energy from wave. These accelerated energetic particles transfer their energy to nonresonant mode through a modulated field. Thus nonresonant plasma wave is amplified at the cost of low frequency resonant mode wave energy. This process occurs if there is a supply of free energy source likes energetic electrons flux or magnetic flux in the system. On ionospheric plasma such a process may play crucial role in generation of plasma instabilities [6].

II. RELATED WORK

In the research paper of Huba et al.[7] ion sound waves are generated mainly at sunrise and sunset duration in the topside ionosphere and not be damped by ion Landau damping for $T_e \gg T_i$ where T_e is electron temperature and T_i is ion temperature. Kantor and Pierce [8] discussed on velocity of charged particles are induced by acoustic wave in the ionospheric region. Pécseli [9] discussed on low frequency electrostatic drift wave turbulence for enhanced density fluctuations in magnetised ionospheric plasma. A number of authors [10, 11, 12] studied ion sound wave under drift wave turbulence in space and laboratory environment. In this paper on the basis of plasma maser effect we present our theoretical investigation on amplification of ion sound wave in presence of low frequency drift turbulence in inhomogeneous ionospheric region.

III. FORMULATION

In this problem we want to study instability of ion sound wave on the basis of weak turbulence theory in the presence of low frequency drift wave turbulent field. We assume along the magnetic field nonresonant electrostatic ion sound wave has propagation vector $\vec{k} = (0, 0, k_{\parallel})$ and propagation vector of the resonant electrostatic drift wave has $\vec{k} = (k_{\perp}, 0, k_{\parallel})$. where

□ and ⊥ mean parallel and perpendicular to the external magnetic field.

For this system, the zero order electron distribution function is considered as [13]

$$f_{oe} = f_{oe}(v_{\perp}^2, v_z) \left[1 + \varepsilon'_e \left(y - \frac{v_x}{\Omega_e} \right) \right] \quad (1)$$

$$\varepsilon'_e = \left(-\frac{1}{f_{oe}} \frac{\partial f_{oe}}{\partial v} \right)_{y=y_o=0} \text{ is electron density gradient.}$$

$$\Omega_e = \frac{eB_o}{mc} \text{ is electron cyclotron frequency.}$$

IV. MATHEMATICAL ANALYSIS

The governing Vlasov-Poisson system of equations for the interaction of ion sound wave with drift wave is given by

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} - \frac{e}{m_e} \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial}{\partial \vec{v}} \right] F(\vec{r}, \vec{v}, t) = 0 \quad (2)$$

and

$$\vec{\nabla} \cdot \vec{E} = -4\pi n_e e [f(\vec{r}, \vec{v}, t) \cdot d\vec{v}] \quad (3)$$

The unperturbed distribution function for electrons and the unperturbed electric field are taken as

$$F_{0e} = f_{oe} + \varepsilon f_{1e} + \varepsilon^2 f_{2e} \quad (4)$$

$$\vec{E}_{0l} = \varepsilon \vec{E}_l$$

Here f_{1e} and f_{2e} are fluctuating parts due to the low frequency drift wave turbulence. We consider wave propagates only in the x-z plan and the system is inhomogeneous in the y-direction. For this Fourier decomposition of physical variable in the y-direction is not considered. Let low frequency electrostatic drift wave has wave field as $\vec{E}_l = (E_{l\perp}, 0, E_{l\parallel})$ with a propagation vector $\vec{k} = (k_{\perp}, 0, k_{\parallel})$ and perturb the quasisteady state by the ion sound wave field $\mu \delta \vec{E}_h$. This acoustic wave having propagation vector $\vec{k} = (0, 0, K_{\parallel})$, electric field $\delta \vec{E} = (0, 0, \delta E_h)$ and frequency Ω . Due to this perturbation, the total perturbed electric field, magnetic field and the electric distribution function are

$$\delta \vec{E} = \mu \delta \vec{E}_h + \mu \varepsilon \delta \vec{E}_{lh} + \mu \varepsilon^2 \Delta \vec{E}$$

$$\delta \vec{B} = 0 \quad (5)$$

$$\delta f_e = \mu \delta f_h + \mu \varepsilon \delta f_{lh} + \mu \varepsilon^2 \Delta f$$

Where δE_{lh} and ΔE are modulating fields, δf_{lh} and Δf are particle distribution functions corresponding to modulating fields and δf_h is fluctuating part due to nonresonant ion sound wave.

Using equation(5) in Vlasov equation(2) for the perturbed state and to the order of μ , we have

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} - \frac{e}{m_e} \left(\vec{E} + \frac{\vec{v} \times \vec{B}_0}{c} \right) \cdot \frac{\partial}{\partial \vec{v}} \right] \delta f_h = \frac{e}{m_e} \delta \vec{E}_h \cdot \frac{\partial}{\partial \vec{v}} f_{oe} \quad (6)$$

To the order of $\mu \varepsilon$, we have

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} - \frac{e}{m_e} \left(\vec{E} + \frac{\vec{v} \times \vec{B}_0}{c} \right) \cdot \frac{\partial}{\partial \vec{v}} \right] \delta f_h = \frac{e}{m_e} \left[\vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} f_h + \delta \vec{E}_h \cdot \frac{\partial}{\partial \vec{v}} f_{1e} + \delta \vec{E}_{lh} \cdot \frac{\partial}{\partial \vec{v}} f_{oe} \right] \quad (7)$$

Using random phase approximation and omitting second order quantities, to the order of $\mu \varepsilon^2$ we have

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} - \frac{e}{m_e} \left(\vec{E} + \frac{\vec{v} \times \vec{B}_0}{c} \right) \cdot \frac{\partial}{\partial \vec{v}} \right] \Delta f = \frac{e}{m_e} \left[\vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} f_{lh} + \delta \vec{E}_{lh} \cdot \frac{\partial}{\partial \vec{v}} f_{1e} \right] \quad (8)$$

To obtain fluctuating parts of the distribution functions, using cylindrical coordinates in velocity space the unperturbed particle orbits along which in order to solve equations (6),(7) and (8) are

$$x' - x = \frac{v_{\perp}}{\Omega_e} \left[\sin \theta - \sin(\theta - \Omega_e \tau) \right]$$

$$y' - y = \frac{v_{\perp}}{\Omega_e} \left[\cos(\theta - \Omega_e \tau) - \cos \theta \right]$$

$$z' - z = v_{\parallel} \tau$$

$$\tau = t' - t$$

To the order of ε from equation (2) we have

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} - \frac{e}{m_e} \left(\frac{\vec{v} \times \vec{B}_0}{c} \right) \cdot \frac{\partial}{\partial \vec{v}} \right] \delta f_{1e} = \frac{e}{m_e} \vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} f_{oe}$$

(9)

Using Fourier transform and the method of characteristics [13] from equation (9) we have

$$f_{1e}(\vec{k}, \omega) = i \sum \frac{e}{m_e} \left[\frac{m}{k_{\perp} T_e} E_{l\perp} \left\{ 1 - \left(k_{\parallel} v_{\parallel} - \omega - \frac{\varepsilon' k_{\perp} T_e}{m \Omega_e} \right) \right\} \Pi_{a,b} \right] f_{oe} - E_{l\parallel} \frac{\partial f_{oe}}{\partial v_{\parallel}} \Pi_{a,b}$$

(10)

Where

$$\Pi_{a,b} = \sum_{a,b} \frac{J_a \left(\frac{v_{\perp} k_{\perp}}{\Omega_e} \right) J_b \left(\frac{v_{\perp} k_{\perp}}{\Omega_e} \right) e^{i(b-a)\theta}}{(a\Omega_e + k_{\perp} v_{\parallel} - \omega) + i0^+}$$

To evaluate the fluctuating part of the distribution function due to high frequency ion sound wave from equation (6) by using Fourier transform and the method of characteristics, we have

$$\delta f_h(\vec{K}, \Omega) = -i \sum \frac{e}{m_e} \delta E_h \frac{\frac{\partial f_{oe}}{\partial v_{\parallel}}}{(K_{\parallel} v_{\parallel} - \Omega)}$$

(11)

Now using Fourier transform and the method of characteristics in equation (7) we obtain

$$\begin{aligned} \delta f_{lh} &= -i \sum \frac{e}{m_e} \frac{\delta E_{lh}}{k_{\perp}} \frac{m_e}{T_e} \left[1 + \left\{ (\Omega - \omega) - (K_{\parallel} - k_{\parallel}) v_{\parallel} - \frac{\varepsilon' T_e k_{\perp}}{\Omega_e m_e} \right\} \right]_{s,t} f_{oe} \\ &+ \sum \left(\frac{e}{m_e} \right)^2 \delta E_h \frac{\partial}{\partial v_{\parallel}} \left[\frac{m_e}{k_{\perp} T_e} E_{l\perp} \left\{ 1 - \left(k_{\parallel} v_{\parallel} - \omega - \frac{\varepsilon' k_{\perp} T_e}{m_e \Omega_e} \right) \right\} \Pi_{a,b} \right] f_{oe} - E_{l\parallel} \frac{\partial f_{oe}}{\partial v_{\parallel}} \Pi_{a,b} \\ &- \sum \left(\frac{e}{m_e} \right)^2 E_{l\perp} \delta E_h \frac{\frac{\partial f_{oe}}{\partial v_{\parallel}}}{(K_{\parallel} v_{\parallel} - \Omega)} \left(\frac{m_e}{k_{\perp} T_e} \right) \left[1 + \left\{ (\Omega - \omega) - (K_{\parallel} - k_{\parallel}) v_{\parallel} - \frac{\varepsilon' T_e k_{\perp}}{m_e \Omega_e} \right\} \right]_{s,t} \end{aligned}$$

(12)

Where

$$\Pi_{s,t} = \sum_{s,t} \frac{J_s \left(\frac{v_{\perp} k_{\perp}}{\Omega_e} \right) J_t \left(\frac{v_{\perp} k_{\perp}}{\Omega_e} \right) e^{i(s-t)\theta}}{(K_{\parallel} - k_{\parallel}) v_{\parallel} - (s\Omega_e + \Omega - \omega)}$$

By using Poisson's equation

$$\vec{\nabla} \cdot \delta \vec{E}_{lh} = -4\pi \sum n_e \int \delta f_{lh} \cdot d\vec{v}$$

(13)

the form of modulated field $\delta \vec{E}_{lh}$ is

$$\delta \vec{E}_{lh} = \Lambda \int [R' - R'' + R''' - R'''] d\vec{v}$$

(14)

Where

$$\begin{aligned} \Lambda &= i \sum \frac{4\pi e n_e}{T |\vec{K} - \vec{k}|} \left(\frac{e}{m_e} \right)^2 \delta E_h \\ \Gamma &= 1 + \sum \frac{\omega_{pe}^2}{|\vec{K} - \vec{k}|^2} \frac{m_e}{T_e} \int \left[1 + \left(\Omega - \omega - \frac{\varepsilon' k_{\perp} T_e}{m_e \Omega_e} \right) \right]_{s,t} f_{oe} \cdot d\vec{v} \\ R' &= \left[-E_{l\perp} \frac{\frac{\partial f_{oe}}{\partial v_{\parallel}}}{K_{\parallel} v_{\parallel} - \Omega} \frac{m_e}{k_{\perp} T_e} \left\{ 1 + \left((\Omega - \omega) - (K_{\parallel} - k_{\parallel}) v_{\parallel} - \frac{\varepsilon' T_e k_{\perp}}{m_e \Omega_e} \right) \right\} \right]_{s,t} \\ R'' &= \left[E_{l\parallel} \frac{\partial}{\partial v_{\parallel}} \left(\frac{\frac{\partial f_{oe}}{\partial v_{\parallel}}}{K_{\parallel} v_{\parallel} - \Omega} \right) \right]_{s,t} \\ R''' &= \frac{\partial}{\partial v_{\parallel}} \left[E_{l\perp} \frac{m_e}{k_{\perp} T_e} \left\{ 1 - \left(k_{\parallel} v_{\parallel} - \omega - \frac{\varepsilon' k_{\perp} T_e}{m_e \Omega_e} \right) \right\} \Pi_{a,b} \right] f_{oe} \Big|_{s,t} \\ R'''' &= \frac{\partial}{\partial v_{\parallel}} \left[E_{l\parallel} \frac{\partial f_{oe}}{\partial v_{\parallel}} \Pi_{a,b} \right]_{s,t} \end{aligned}$$

Here ω_{pe} is electron plasma frequency.

The electric field for high frequency ion sound wave is obtained by Poisson's equation

$$\vec{\nabla} \cdot \delta \vec{E}_h = -4\pi \sum n_e \int [\delta f_h(K, \Omega) + \Delta f(K, \Omega)] \cdot d\vec{v}$$

(15)

Here perturbed part of distribution function for the modulating field is

$$\begin{aligned} \Delta f_e(\vec{K}, \Omega) &= \sum \frac{e}{m_e - \alpha} \int_0^{\Omega} \left(\vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} \delta f_{lh} + \delta \vec{E}_{lh} \cdot \frac{\partial}{\partial \vec{v}} \delta f_{1e} \right) e^{i[\vec{K} \cdot (\vec{r}' - \vec{r}) - \Omega \tau]} d\tau \\ &= \Lambda_1 + \Lambda_2 \end{aligned}$$

(16)

Where

$$\Delta_1 = \sum \frac{e}{m_e - \alpha} \int_0^{\infty} \vec{E}_{lh} \cdot \frac{\partial}{\partial \vec{v}} \delta f_{1h} e^{[i\{\vec{K} \cdot (\vec{r}' - \vec{r}) - \Omega \tau\}]} d\tau$$

$$= -\sum \frac{ie}{m_e} \left[E_{l\perp} \frac{\partial}{\partial v_{\perp}} + E_{l\parallel} \frac{1}{(K_{\parallel} v_{\parallel} - \Omega)} \frac{\partial}{\partial v_{\parallel}} \right] \delta f_{1h} \quad (17)$$

$$\Delta_2 = \sum \frac{e}{m_e - \alpha} \int_0^{\infty} \vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} \delta f_{1e} e^{[i\{\vec{K} \cdot (\vec{r}' - \vec{r}) - \Omega \tau\}]} d\tau$$

$$= -\sum \frac{ie}{m_e} \left[\frac{\delta E_{lh}}{|\vec{K} - \vec{k}|} \left\{ \frac{\partial}{\partial v_{\perp}} - \frac{1}{(K_{\parallel} v_{\parallel} - \Omega)} k_{\parallel} \frac{\partial}{\partial v_{\parallel}} \right\} f_{1e} \right] \quad (18)$$

For the nonlinear coupling between resonant and nonresonant waves, a high frequency nonlinear force is developed and which acts on electrons. According to electrodynamics these accelerated electrons can radiate an amplified nonresonant wave. Nambu et al.[14] and Barash et al.[15] defined this electron accelerated nonlinear force as the combination of direct coupling term \vec{F}_1 and polarization coupling term \vec{F}_2 respectively and expressed as

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$= n_e e \int \left\langle \vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} \delta f_{1h} \right\rangle \vec{v} d\vec{v} + n_e e \int \left\langle \vec{E}_h \cdot \frac{\partial}{\partial \vec{v}} \delta f_{1e} \right\rangle \vec{v} d\vec{v} \quad (19)$$

From equation (8) and applying Fourier transform and method of characteristics, the expression of z-component of non linear force acting on unit volume of particles is

$$F_z(\vec{K}_1, \Omega) = F_{z1}(\vec{K}_1, \Omega) + F_{z2}(\vec{K}_2, \Omega) \quad (20)$$

Where

$$F_{z1}(\vec{K}_1, \Omega) = -en_0 \int \frac{\Omega}{\Omega - K_{\parallel} v_{\parallel}} \left\langle \vec{E}_l \cdot \frac{\partial}{\partial \vec{v}} \delta f_{1h} \right\rangle v_{\parallel} dv \quad (21)$$

$$F_{z2}(\vec{K}_2, \Omega) = -en_0 \int \frac{\Omega}{\Omega - K_{\parallel} v_{\parallel}} \left\langle \vec{E}_h \cdot \frac{\partial}{\partial \vec{v}} \delta f_{1e} \right\rangle v_{\parallel} dv \quad (22)$$

Here we have used

$$\int_{-\alpha}^0 e^{[i\{K_{\parallel}(z' - z) - \Omega \tau\}]} d\tau = -\frac{i}{(K_{\parallel} v_{\parallel} - \Omega)}$$

In earlier works [10,11,12] found that direct coupling term plays a dominant role for the growth of ion sound wave. Here we neglect polarization coupling term and consider

direct coupling term only for the role of growth of electrostatic ion sound wave.

Considering the plasma maser effect arises under the condition $\omega - k_{\parallel} v_{\parallel} = 0$, $\Omega < K_{\parallel} v_{\parallel}$ and to consider a=b=s=t=0 for taking dominant terms only, after lengthy straight forward calculation we obtain

$$F_{z1}(K, \Omega) = \sqrt{\pi} n_e \Omega^2 \delta E_h \left(\frac{e}{m_e} \right)^2 E_{l\parallel} \left(E_{l\perp} \frac{e'}{\Omega_e} + 2E_{l\parallel} \frac{v_d}{v_e^2} \right) \frac{1}{v_e v_d^2} \frac{1}{|k_{\parallel} K_{\parallel}^2} e^{-\left(\frac{v_d}{v_e} \right)^2} \quad (23)$$

where $v_d = \frac{\omega}{k_{\parallel}}$ is the drift velocity.

Based on the method of chen[16] applying the electron continuity equation, the electron fluid equation and using Fourier transforms and linearising about the steady state, after calculation we obtain the dispersion relation of ion sound wave in presence of drift wave turbulence as

$$\Omega = \chi^2 \left(\frac{k_B T_e}{m_e} \right) + i \frac{\chi}{m_e} \frac{F_{z1}}{\delta n_{e0}} \quad (24)$$

where δn_{e0} is first order electron density fluctuation.

After neglecting nonlinear frequency shift ,real frequency of ion sound wave from equation (24) we obtain

$$\Omega_r = \chi \sqrt{\frac{k_B T_e}{m_e}} \quad (25)$$

and the growth rate of ion sound wave

$$\gamma = \frac{\chi F_{z1}}{2m_e \Omega_r \delta n_{e0}} \quad (26)$$

Here first order perturbation in electron density from Boltzmann relation after neglecting higher order terms is

$$\delta n_{e0} = n_0 \frac{e \phi_1}{k_B T_e} \quad (27)$$

where ϕ_1 is first order potential fluctuation.

From equation (26) we obtain

$$\frac{\gamma}{\Omega} = \frac{\sqrt{\pi}}{2} \left(\frac{e}{m_e} \right)^2 \frac{\Omega}{m_e k_B T_e} \sqrt{\frac{k_B T_e}{m_e}} E_{l\parallel} \left(E_{l\perp} \frac{e'}{\Omega_e} + 2E_{l\parallel} \frac{v_d}{v_e^2} \right) \frac{1}{v_e v_d^2} \frac{1}{|k_{\parallel} K_{\parallel}^2} e^{-\left(\frac{v_d}{v_e} \right)^2} \quad (28)$$

Which is the expression of growth rate of ion sound wave for the direct coupling with drift turbulence in inhomogeneous plasma system.

For the weak density gradient in ionospheric plasma we can take $\varepsilon' = 0$ equation (28) become

$$\frac{\gamma}{\Omega} = \sqrt{\pi} \left(\frac{e}{m_e} \right)^2 \frac{\Omega}{m_e} k_B T_e \sqrt{\frac{k_B T_e}{m_e}} E_{\perp}^2 \frac{1}{v_e^3 v_d} \frac{1}{|k_{\perp} K_{\perp}^2|} e^{-\left(\frac{v_d}{v_e} \right)^2} \quad (29)$$

V.RESULT AND CONCLUSION

Based on weak turbulence theory we are investigated the growth rate of the nonresonant ion sound wave in the presence of resonant drift wave through plasma maser instability. To study nonlinear wave-particle interaction the electron continuity equation and the electron fluid equation are used in the Vlasov-Poisson system of mathematical frame. For the amplification of high frequency ion sound wave in the presence of universal drift wave turbulence the nonlinear force act as a driving force in the inhomogeneous ionospheric plasma. From observational data from ROSE rocket flight [17] and other sources [18,19] the plasma parameters in ionospheric region and typical parameters of ion sound wave and drift wave in space are

$$T_e = 400 K, E_{\perp} = 4 \times 10^{-2} Vm^{-1}, k_{\perp} \approx 10^{-5} m^{-1}$$

$$K_{\perp} \approx 10^{-5} m^{-1}, \Omega \approx \Omega_e \approx 10^6 s^{-1}, v_d = 10^6 ms^{-1}, v_e = 4.19 \times 10^6 ms^{-1}$$

The estimated growth rate for weak density gradient in ionospheric plasma from equation (29) is obtained as

$$\frac{\gamma}{\Omega} = 10^{-1}$$

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