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Mapping of Nambu-Goto String in d Space-time Dimension and QCD Potential

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Abstract- The bosonic string potential model proposed by Nambu and Goto contains Coulombic Lüscher term, which is a function of the space-time dimension 'd'. In this work, we report some novel features of QCD-String mapping, if the colour-cum-flavour dependent coefficient of the QCD coulomb potential and the Luscher coefficient in d space-time dimension are conjectured to be equivalent.

Keywords : Nambu-Goto string, Luscher term, QCD. PACS Nos: 12.39.-x , 12.39.Jh, 12.39.Pn.

I. INTRODUCTION

Inter quark potential is mainly generated by gluon dynamics, under static approximation, with valence quark effect. At distances smaller than 1 Fermi, the quark-antiquark potential is coulombic, due to Asymptotic Freedom. At large distances the potential should be linear due to formation of a confining flux tube [1]. When these tubes are much longer than their thickness we can describe them by semi-classical Nambu strings [2]. This explains the existence of approximately linear Regge trajectories [3]. Empirical evidence for a string-like structure of hadrons comes from arranging mesons and baryons into approximately linear Regge trajectories.

Throughout its history, string theory has been intertwined with the theory of strong interactions. The AdS/CFT [4] correspondence succeeded in making precise connections between 4-dimensional gauge theories and higher dimensional string theory.

In recent years, there has been growing interest in the possible correlation between QCD and string theory through ACD/QCD approach [5]. Inspired by such approach, we have analyzed some novel features of QCD-String correspondence, in this paper.

The strength of the quark-gluon interaction is characterized by the coupling constant $\alpha_s(Q^2)$[9], defined by convention in a particular dimensional regularization scheme. The coulombic term of QCD potential contains this colour-cum-flavour dependent coupling constant.

In this paper, The Nambu-Goto string potential [6] and standard QCD potential [7] are compared. The colour-cum-flavour dependent coefficient of the QCD coulomb potential and the Lüscher term of Nambu-Goto potential in d space-time dimension are conjectured to be equivalent, and the possible QCD-string correspondence is studied. The correspondence of QCD parameters like Q^2, N_c and N_f with the space-time dimension 'd' of string model is also reviewed.

Chapter 2 contains brief review of formalism. In chapter 3 we report our conjectured correspondence and chapter 4 contains conclusion and comments.

II. FORMALISM

2.1 Running coupling constant in QCD:

In the framework of perturbative QCD (pQCD) [8], predictions for observables are expressed in terms of renormalized coupling [9] $\alpha_s(\mu^2)$, a function of an renormalization scale μ. When μ is close to the scale of momentum transfer Q in a given process, $\alpha_s(Q^2)$ is a measure of the effective strength of the strong interaction in the process.

This running coupling $\alpha_s(Q^2)$ satisfies the Renormalisation Group Equation (RGE) [10]:

$$Q^2 \frac{d\alpha_s}{dQ^2} = \beta(\alpha_s) = -(b_0\alpha_s^2 + b_1\alpha_s^3 + b_2\alpha_s^4 + \ldots) \quad (1)$$

where,

$$b_0 = \frac{11C_A - 4N_f T_R}{12\pi} \quad (2)$$

This is the one-loop beta function coefficient and

$$b_1 = \frac{17C_A^2 - N_f T_R(10C_A + 6C_F)}{24\pi^2} \quad (3)$$

This is the two-loop beta function coefficient.

Here, T_R is an invariant in QCD colour algebra [11] and is conventionally taken to be ½. $C_A = N_c$.

C_F is called the colour factor which is given by :

$$C_F\, N = tr \sum_{a,b} \lambda^a \lambda^b = \delta^{a,b} T_R \delta_{a,b} = \frac{N_c^2 - 1}{2} \quad (6)$$

This gives, $C_F = \frac{N_c^2-1}{2N_c}$ (7)

From equation (1), the perturbative expansion of running coupling is obtained as [12]:

$$\alpha_s(Q^2) = \frac{1}{b_0 t}\left(1 - \frac{b_1}{b_0^2}\frac{\ln t}{t} + \cdots\right) \quad (8)$$

with $t = \ln\frac{Q^2}{\Lambda^2}$

Λ defines the scale where the perturbatively-defined coupling would diverge, i.e., it is the non-perturbative scale of QCD[13].

2.2 QCD and String Potentials:

It has been found that the most consistent inter-quark potential in QCD has the form[6] :

$$V(r) = -\frac{C_F\alpha_s}{r} + br + c \quad (9)$$

This formula is basically an interpolation between the long range non-perturbative (br) and short range repulsive $(-\frac{C_F\alpha_s}{r})$ parts of the force between pairs of quarks.

Here the coefficient $-C_F\alpha_s$ of the coulombic term depends upon the colour (N_c) and flavor (N_f) quantum numbers and also on momentum transfer Q^2.

Again, from Wilson loop calculation, it is now established that the static quark-antiquark potential V(r) for bosonic strings, asymptotically rises linearly with r as [14]:

$$V(r) \underset{r\to\propto}{\longrightarrow} \sigma r \quad (10)$$

This is the linear quark confinement. The generally accepted leading order correction to this quark-antiquark linear potential has been calculated as [2]: $-\frac{\pi(d-2)}{24\, r}$

Considering this first order correction, the Nambu-Goto string potential can be expressed as [7] :

$$V(r) = \frac{\gamma}{r} + \sigma r + \mu_0 \quad (11)$$

Here $\frac{\gamma}{r}$ is the universal Lüscher term with $\gamma = -\frac{\pi(d-2)}{24}$ [15]. It does not depend upon the gauge group - depending only on the space-time dimension d; it is universal in the sense that its value is the same for a large class of string action. σ is the string tension, which characterizes the strength of the confining force, its value is 0.89 GeV/fm (~ 0.178 GeV^2). μ_0 is a regularisation constant.

III. CONJECTURED CORRESPONDENCE

We compare the Nambu-Goto string potential in equation (11) with the standard QCD potential of equation (9). In standard QCD potential, the first term is perturbative and coulombic, while the similar term in equation (11) is having non-perturbative origin. The standard value of b [16] in equation (9) is 0.183 Gev^2 (~ 0.915 GeV/fm) to be compared with the corresponding value of σ [17] as 0.178 GeV^2 (~ 0.89 GeV/fm). The constant μ_0 is a regularisation constant in Nambu-Goto string potential, whereas that scale factor 'c' in equation (9) is assumed to have numerical value of 1 GeV [18], to make it compatible with the usual masses of the mesons.

Further, we would like to study the correspondence between coefficient of coulombic term in QCD potential and the Lüscher coefficient in Nambu-Goto potential. The conjectured equivalence of Lüscher coefficient and the coefficient of coulombic term in QCD potential leads to :

$$\frac{\pi(d-2)}{24} = C_F\alpha_s(Q^2) \quad (12)$$

with

$$C_F = \frac{N_c^2-1}{2N_c} \quad (13)$$

At leading order (LO), the running coupling expansion is given by [12]

$$\alpha_s(Q^2) = \frac{1}{b_0 t}, \quad \text{with } t = \ln\frac{Q^2}{\Lambda^2} \quad (14)$$

b_0 is given by equation (2). From equation (12) and (14) , we obtain correspondence at LO as:

$$\frac{\pi(d-2)}{24} = \frac{N_c^2-1}{2N_c}\,\frac{12\pi}{11N_c - 2N_f}\,\frac{1}{t} \quad (15)$$

The next-to-leading order (NLO) of running coupling expansion [12] is,

$$\alpha_s(Q^2) = \frac{1}{b_0 t}\left(1 - \frac{b_1}{b_0^2}\frac{\ln t}{t}\right) \quad (16)$$

with b_1 given by equation (3).

Equations (12) and (16) give the correspondence at NLO as:

$$\frac{\pi(d-2)}{24} = \frac{N_c^2-1}{2N_c}\,\frac{12\pi}{11N_c-2N_f}\,\frac{1}{t}\left(1 - \frac{6[17N_c^2 - N_f T_R \frac{1}{2}(10N_c + 6C_F)]}{(11N_c - 2N_f)^2}\,\frac{\ln t}{t}\right) \quad (17)$$

Equations (15) and (17) are the conjectured correspondence between coulombic term of QCD potential and Lüscher term in Nambu-Goto potential. These two equations immediately show that while d=2 corresponds to $\alpha_s = 0$ as $Q^2 \to \infty$ - the force field limit of QCD, $d \to \infty$ corresponds to $Q^2 = \Lambda^2$ - the non-perturbative scale. Thus, $2 < d$ is the allowed limit for the space-time dimension, if the conjectured correspondence is taken seriously.

In table-1, we study the conjectured correspondence between the dimension of the string and the virtuality Q^2 explored in QCD : for $d \leq 26$, taking N_c=3 & N_f =3,6 with $\Lambda =$ 300MeV. Q^2 vs d graph at LO and LNO for N_c = 3 and N_f = 3 are shown in figures 1.

It shows that strings of a definite dimension have correspondence with QCD at a definite Q^2. Here, since dimension, by definition, is positive integer, such correspondence is possible only for discrete values of Q^2 explored in QCD. Further, it is evident from table-1 and also from figures-1 that beyond d=5, the NLO effect in α_s becomes insignificant.

Table-1

d	Q^2 (in GeV^2) at LO (Eqn. 15)		Q^2 (in GeV^2) at NLO (Eqn. 17)	
	For N_f = 3	For N_f = 6	For N_f = 3	For N_f = 6
2	∞	∞	∞	∞
3	1.352 x 10^5	7.864 x 10^6	1.301 x 10^4	1.527 x 10^6
4	110.296	841.269	18.951	235.872
6	3.151	8.701	1.124	3.694
8	0.963	1.896	0.527	1.049
10	0.532	0.885	0.375	0.589
15	0.269	0.367	0.258	0.319
20	0.196	0.249	0.217	0.247
25	0.167	0.199	0.196	0.214
26	0.163	0.193	0.193	0.209

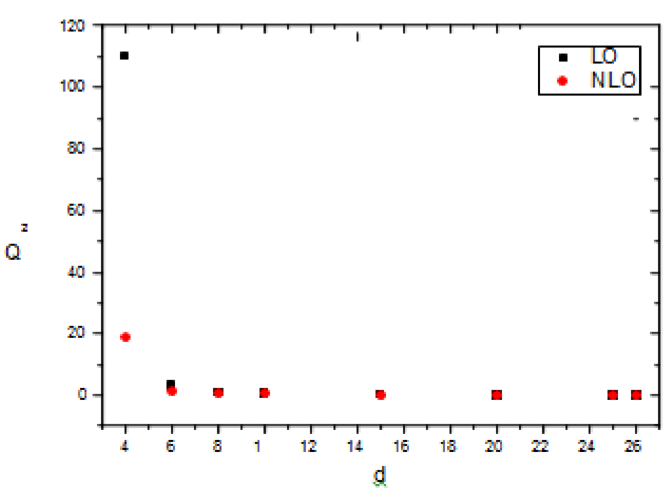

Fig-1: Q^2 vs d with N_c=3 and N_f=3

IV. CONCLUSION AND COMMENTS

In this paper, we have showed that, a conjectured mapping between Lüscher term of Nambu-Goto Strings and QCD standard potential yields relationship between space-time dimension d in string theory and virtuality Q^2 explored in QCD. Here, since dimension d, by definition, is positive integer, such mapping is limited to a specific set of virtuality. For a one to one mapping, the dimension of the strings needs to be continuous or fractional – a notion, perhaps too novel to be taken seriously at least at the present manner. Feasibility of calculative potentiality of such a mapping and its possible relationship with celebrated ADS/QCD [5] correspondence needs further study.

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