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Fluid Dynamical Instabilities in Magnetized Partially Ionized Dense Dusty Plasma

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Abstract — Fluid dynamical instabilities in magnetized partially ionized dense dusty plasma are studied by taking into account relative flow between dust and neutral gas. Following Hurwitz criterion, the onset criteria for instabilities are derived for different densities of the neutral gas and dust components across the interface. It is found that in case of no significant magnetic field stabilization occurs not only due to dust neutral gas collisions but due to relative flow also. Our result might be useful in many situations of astrophysical magnetized dusty plasma namely comets and circumsteller dusty disk e.g. T-Tauri stars.

Keywords—*Dusty Plasma, Partially Ionized Plasma, Instabilities*

I. INTRODUCTION

Instabilities are ubiquitous in partially ionized dusty plasmas. The study of instabilities in partially ionized dusty plasmas has drawn considerable interest in recent past in many different astrophysical contexts such as accretion disks [1] and relativistic jets [2]. The Rayleigh-Taylor instability may be responsible for the formation of waves and bubbles in the Earth's equatorial region [3,4]. D'Angelo [5] studied the Rayleigh-Taylor instability in such a dusty plasma where the dust grains have been assumed to be massive. He investigated the effects of negatively and positively charged dust grains on the gravitational Rayleigh-Taylor instability and found that negatively charged dust have stabilizing effects. A flute like instability which is different from the usual Rayleigh-Taylor instability is investigated by Varma and Shukla [6]. Another important instability, which occurs when adjacent layers of fluid are in relative motion, is called Kelvin-Helmholtz instability and has been analyzed for a conductive magnetized incompressible fluids streaming along the direction of the magnetic field[7].

Goertz [8] described various phenomena and instabilities occurring in dusty plasma of solar system in his review paper. Shear flows play an important role in the dynamics of partially ionized dusty plasma because they induce the unstable Kelvin-Helmholtz modes in various physical situations namely, superwinds of primeval galaxies in the intergalactic medium [9] and the amplification of self induced magnetic fields in the early Universe [10]. The existence of fluid dynamical instabilities for the partially ionized flow have been discussed by Kamaya and Nishi [11]. They found that the instability of the Alfve'n wave for any n and the two fluid instability for any $'k'$ if $n=1$. The Alfven instability appears when its wave number is smaller than a critical value.

Birk [12] derived criteria for unstable Rayleigh-Taylor modes in partially ionized dusty plasma for different density characteristics of the neutral gas and dust components across the interface and found that dust-neutral gas collisions limit the range of unstable wavelengths. Shear flow instabilities in magnetized partially dense dusty plasma have been studied by Birk and Wiechen [13]. They derived onset criteria for instabilities with and without electrical resistivity and found that momentum exchange between the dust and neutral gas stabilized long wavelength perturbations. Excited unstable modes lead to the formation of current sheets and vortices.

In the present paper, we study the fluid dynamical instabilities in magnetized partially ionized dense dusty plasma by taking relative flow between dust and neutral gas. The plan of the paper is as follows. In section 2, the problem is formulated in terms of basic equations governing the motion. Instabilities criteria are obtained and compared with previous studies in section 3.

II. THEORY AND BASIC EQUATIONS

We consider a magnetized partially ionized dense dusty plasmas whose dynamics are governed by dust and neutral gas components. The dynamics of dusty plasma are characterized by collective behavior for the parameter regime of ordering $a << d << \lambda_d$, where a, d and λ_d are the dust grain radius, the average inter-grain distance and the plasma Debye length. The electrons dynamics is not considered as the electrons have no significant influence on the overall behaviuor of dusty plasma. The plasma is considered quasineutral. So we have

$$n_i z_i = n_d z_d + n_e \tag{1}$$

where n and z are number density and charge number and the suffixes i, d, e denote ion, dust and electron fluids respectively. Since the assumption of incompressibility holds very well for the perturbations with velocity amplitudes well below the dust-neutral gas sound velocities with propagation time scales larger than sound time scales. The dynamics of fluids are taken incompressible $(\nabla.\mathbf{v_d} = \nabla.\mathbf{v_n} = 0)$. The relevant equations governing the motion of dense dusty plasma are as follows [12]:

$$\frac{\partial \rho_d}{\partial t} + (\mathbf{v_d}.\nabla)\rho_d = 0 \tag{2}$$

$$\frac{\partial \rho_n}{\partial t} + (\mathbf{v_n}.\nabla)\rho_n = 0 \tag{3}$$

Here $\rho_{d,n}$ are the dust and neutral gas densities and $\mathbf{v_{d,n}}$ are their velocities. The momentum equations for the charged components of the fluid dusty plasma where the ion and electron inertia are negligible in the considered dense dusty plasma, and neutral gas are given by

$$\rho_d \frac{\partial v_d}{\partial t} + (\mathbf{v_d}.\nabla)\mathbf{v_d} = \nabla(p_d + p_i + p_e) + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} - \rho_d \nu_{dn}(\mathbf{v_d} - \mathbf{v_n}) + \rho_d \mathbf{g} \tag{4}$$

$$\rho_n \frac{\partial \mathbf{v_n}}{\partial t} + (\mathbf{v_n}.\nabla)\mathbf{v_n} = -\nabla p_n + \rho_d \nu_{dn}(\mathbf{v_d} - \mathbf{v_n}) + \rho_n \mathbf{g} \tag{5}$$

where $p_{d,i,e,n}$ are the dust, ion and neutral gas pressures. The symbols $\mathbf{B}, \mathbf{g}$ and ν_{dn} are the magnetic field, the gravitational acceleration and the effective elastic collision frequency between the dust and neutral gas particles respectively. The electron partial pressure is usually negligible in the total pressure of charged components $p_c = p_d + p_i + p_e$. The magnetic induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v_d} \times \mathbf{B}) \tag{6}$$

The resistivity, Hall effect as well as other small effect of magnetic field generation are not considered.

We consider equilibrium state where the homogeneous magnetic field $\mathbf{B_0}$ is taken along the z-axis and the homogeneous gravitational field $\mathbf{g} = -g\hat{e}_y$ We consider interface along the z-axis i.e along the equilibrium flow and magnetic field. The other equilibrium quantities on either side of the interface are of the form

$$\rho_{d0} = \begin{cases} \rho_{dI}, & y > 0, \\ \rho_{dII}, & y < 0, \end{cases} \tag{7}$$

$$\rho_{n0} = \begin{cases} \rho_{nI}, & y > 0, \\ \rho_{nII}, & y < 0, \end{cases} \tag{8}$$

III. DERIVATION OF ONSET CRITERIA

A constant relative velocity of dust to the neutral gas is assumed $\mathbf{v_0}$ with motion $\mathbf{v_{d0}}$ in dust and $\mathbf{v_{n0}}$ in neutral gas and $\mathbf{v_{d0}} \neq \mathbf{v_{n0}}$ is taken along the z-axis and is such that

$$\mathbf{v_{d0}} - \mathbf{v_{n0}} = \mathbf{v_0} \tag{9}$$

Let $\mathbf{v}', \mathbf{B}', \rho'$ and p' denote the perturbed quantities for velocity, magnetic field, density and pressure due to a small disturbance to the system. Linearizing equations (1)-(6) about the equilibrium, we obtain

$$\frac{\partial \rho_d'}{\partial t} + (\mathbf{v_{d0}}.\nabla)\rho_d' = 0 \tag{10}$$

$$\frac{\partial \rho_n'}{\partial t} + (\mathbf{v_{n0}}.\nabla)\rho_n' = 0 \tag{11}$$

$$\rho_{d0}\left(\frac{\partial \mathbf{v_d'}}{\partial t} + \left(\mathbf{v_{d0}}.\nabla\right)\mathbf{v_d'}\right) = -\nabla p_c' + \frac{1}{4\pi}\left(\nabla \times \mathbf{B}'\right) \times \mathbf{B_0} - \rho_0 \nu_{dn}\left(\mathbf{v_d'} - \mathbf{v_n'}\right) + \rho_d' \nu_{dn}\left(\mathbf{v_{d0}} - \mathbf{v_{n0}}\right) + \rho_d' \mathbf{g} \tag{12}$$

$$\rho_{n0}\left(\frac{\partial \mathbf{v_n'}}{\partial t} + \left(\mathbf{v_{n0}}.\nabla\right)\mathbf{v_n'}\right) = -\nabla p_n' + \rho_{d0}\nu_{dn}\left(\mathbf{v_d'} - \mathbf{v_n'}\right) + \rho_d' \nu_{dn}\left(\mathbf{v_{d0}} - \mathbf{v_{n0}}\right) + \rho_n' \mathbf{g} \tag{13}$$

$$\frac{\partial \mathbf{B}'}{\partial t} = \nabla \times \left(\mathbf{v_d'} \times \mathbf{B_0}\right) + \nabla \times \left(\mathbf{v_{d0}} \times \mathbf{B}'\right) \tag{14}$$

Fourier analyzing the perturbations by taking the following form

$$\Psi(x, y, z, t) = \Psi(y)\exp(\omega t + ik_x x + ik_z z) \tag{15}$$

where Ψ represents any of the perturbed field quantities of the considered dusty plasma, ω is the frequency, and k_x and k_z are the wavenumbers along $x-$ and $z-$ axes, we obtain

$$-ik_x \frac{B_0^2}{4\pi}\frac{\partial v_{dy}}{\partial y} - \left[\rho_{d0}\left(\tilde{\omega} - v_0 k_z\right)^2 - k^2 \frac{B_0^2}{4\pi} + i\rho_{d0}\nu_{dn}\left(\tilde{\omega} - v_0 k_z\right)\right] v_{dx} + i\rho_{d0}\nu_{dn}\left(\tilde{\omega} - v_0 k_z\right) v_{nx} + k_x p_c\left(\tilde{\omega} - v_0 k_z\right) = 0 \tag{16}$$

$$\left[-i\left(\tilde{\omega} - v_0 k_z\right)^2 \rho_{d0} + i\frac{B_0^2}{4\pi}k_z^2 + \nu_{dn}\left(\tilde{\omega} - v_0 k_z\right)\rho_{d0} - ig\frac{\partial \rho_{d0}}{\partial y}\right] v_{dy} - i\frac{B_0^2}{4\pi}\frac{\partial^2 v_{dy}}{\partial y^2} + k_x \frac{B_0^2}{4\pi}\frac{\partial v_{dx}}{\partial y} - \nu_{dn}\left(\tilde{\omega} - v_0 k_z\right)\rho_{d0} v_{ny} + \left(\tilde{\omega} - v_0 k_z\right)\frac{\partial p_c}{\partial y} = 0 \tag{17}$$

$$\rho_{d0}(\tilde{\omega}-v_0k_z)\left[v_{dn}-i(\tilde{\omega}-v_0k_z)\right]\frac{\partial v_{dy}}{\partial y}+k_x\rho_{d0}(\tilde{\omega}-v_0k_z)\left[(\tilde{\omega}-v_0k_z)+iv_{dn}\right]v_{dx}-\rho_{d0}v_{dn}$$
$$(\tilde{\omega}-v_0k_z)\frac{\partial v_{ny}}{\partial y}-v_{dn}k_zv_0\frac{\partial\rho_{d0}}{\partial y}v_{dy}-i(\tilde{\omega}-v_0k_z)v_{dn}k_x\rho_{d0}v_{nx}+(\tilde{\omega}-v_0k_z)k_z^2p_c=0 \tag{18}$$

$$(-\rho_{n0}\tilde{\omega}-i\rho_{d0}v_{dn})v_{nx}+i\rho_{d0}v_{dn}v_{dx}+k_xp_n=0 \tag{19}$$

$$\left(-i\tilde{\omega}^2\rho_{n0}+\rho_{d0}v_{dn}\tilde{\omega}-ig\frac{\partial\rho_{n0}}{\partial y}\right)v_{ny}-\rho_{d0}v_{dn}\tilde{\omega}v_{dy}+\tilde{\omega}\frac{\partial p_n}{\partial y}=0 \tag{20}$$

$$\left[-i\rho_{n0}\tilde{\omega}(\tilde{\omega}-v_0k_z)+\rho_{d0}v_{dn}(\tilde{\omega}-v_0k_z)\right]\frac{\partial v_{ny}}{\partial y}+\left[k_x\tilde{\omega}(\tilde{\omega}-v_0k_z)\rho_{n0}+ik_x\rho_{d0}v_{dn}(\tilde{\omega}-v_0k_z)\right]v_{nx}$$
$$-\rho_{d0}v_{dn}(\tilde{\omega}-v_0k_z)\frac{\partial v_{dy}}{\partial y}+v_{dn}k_zv_0\frac{\partial\rho_{d0}}{\partial y}v_{dy}-i\rho_{d0}k_xv_{dn}(\tilde{\omega}-v_0k_z)v_{dx}+k_z^2(\tilde{\omega}-v_0k_z)p_n=0 \tag{21}$$

where $k^2=k_x^2+k_z^2$ and $\tilde{\omega}=i\omega-k_zv_{n0}$ is Doppler-shifted complex frequency.

We assume that surface perturbations $v_{d,ny}$ decay exponentially as $v_{d,ny}=v_{d,ny}(0)\exp(\pm k_yy)$ as $y\to\pm\infty$. We take $v_{d,nx},v_{d,nz},\frac{\partial v_{d,ny}}{\partial y}$ and $p_{d,n}$ are odd functions of y for the surface perturbations. Hence, $\left[\tilde{\omega}^2\rho_{d0}v_{dx}\right]=\tilde{\omega}^2\bar{\rho}_{d0}[v_{dx}]$, $\left[\tilde{\omega}^2\rho_{d0}\,\partial v_{dy}/\partial y\right]=-2k_y\tilde{\omega}^2\bar{\rho}_{d0}v_{dy}(0)$, and $\int_I^{II}\tilde{\omega}\,\partial\rho/\partial y\,dy=\tilde{\omega}[p_c]$, where bracket $[\;]$ denotes the jump across the interface $(y=0)$ and an overline on physical quantity represents the mean value of that quantity as $\phi=\tfrac{1}{2}(\phi_I+\phi_{II})$. From equations (16) - (21), the coefficient determinant C of $iv_{dy}(0),[v_{dx}],\tilde{\omega}[p_c],v_{ny}(0),[v_{nx}],\tilde{\omega}[p_n]$ is given by

$$C=\begin{vmatrix} M & R & k_x & 0 & i\Omega_1 & 0 \\ \zeta & k_x\frac{B_0^2}{4\pi} & 1 & 0 & 0 & 0 \\ 2k_yD & k_xD & k_z^2 & 2k_y\Omega_1 & -ik_x\Omega_1 & 0 \\ 0 & i\bar{\rho}_{d0}v_{dn} & 0 & 0 & -A & k_x \\ 0 & 0 & 0 & -i[\rho_{n0}]g & 0 & \tilde{\omega} \\ -2ik_y\Omega_1 & -ik_x\Omega_1 & 0 & 2k_yF_1 & k_xF_2 & k_z^2\tilde{\omega}_1 \end{vmatrix} \tag{22}$$

where

$$\tilde{\omega}_1 = \tilde{\omega} - k_z v_0, \quad \Omega = v_{dn}\tilde{\omega}\bar{\rho}_{d0}, \quad \Omega_1 = v_{dn}\tilde{\omega}_1\bar{\rho}_{d0}$$

$$A = -\tilde{\omega}\bar{\rho}_{n0} - i\bar{\rho}_{d0}v_{dn}, \qquad \zeta = -g\left[\rho_{d0}\right] + k_y \frac{B_0^2}{2\pi}$$

$$D = \tilde{\omega}_1^2\bar{\rho}_{n0} + i\Omega_1, \qquad M = k_x k_y \frac{B_0^2}{2\pi}, \qquad R = k^2\frac{B_0^2}{4\pi} - D$$

$$F_1 = i\tilde{\omega}\tilde{\omega}_1\bar{\rho}_{n0} - \Omega_1, \qquad F_2 = \tilde{\omega}\tilde{\omega}_1\bar{\rho}_{n0} + i\Omega_1$$

The determinant $C = 0$ gives the following characteristic complex polynomial $\omega^8 + (a_1 + ib_1)\omega^7 + (a_2 + ib_2)\omega^6 + (a_3 + ib_3)\omega^5 + \cdots\cdots + (a_8 + ib_8) = 0$ of eighth order in ω. To discuss the instability of the system, we follow the Hurwitz criterion described by Hurwitz [14] and Giaretta [15], and construct the test sequence $h_0 = 1,\ h_1 = a_1 \cdots\cdots$

$$h_r = (-1)^{r(r-1)/2}\begin{vmatrix} a_1 & a_0 & 0 & 0 & 0 & \cdot & \cdot \\ -b_2 & -b_1 & a_1 & a_0 & 0 & \cdot & \cdot \\ a_3 & a_2 & b_2 & b_1 & a_1 & \cdot & \cdot \\ -b_4 & -b_3 & a_3 & a_2 & -b_2 & \cdot & \cdot \\ a_5 & a_4 & b_4 & b_3 & a_3 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{2r-1} & a_{2r-2} & b_{2r-2} & b_{2r-3} & a_{2r-3} & \cdot & \cdot \end{vmatrix}. \tag{23}$$

for the above complex polynomial. The condition for the instability is that any one of the h_i's be negative. It can be obtained from the third Hurwitz sub-determinant

$$h_2 = (-1)\begin{vmatrix} a_1 & 1 & 0 \\ -b_2 & -b_1 & a_1 \\ a_3 & a_2 & b_2 \end{vmatrix} < 0 \tag{24}$$

Hence the instability condition derived from equation (24) is given by

$$g(\rho_{II} - \rho_I) > \frac{k_y}{k^2}\left(k_z^2\frac{B_0^2}{2\pi} + (\rho_I + \rho_{II})\left(8v_{dn}^2 + \frac{1}{2}k_z^2 v_0^2\right)\right) \tag{25}$$

If we consider that there is no relative velocity v_0 i.e $v_{d0} = v_{n0}$ then the instability condition reduces to

$$g(\rho_{II} - \rho_I) > \frac{k_y}{k^2}\left(k_z^2\frac{B_0^2}{2\pi} + 8(\rho_I + \rho_{II})v_{dn}^2\right) \tag{26}$$

This condition for instability is same to that condition (15) obtained by Birk [12]. It can be observed from condition (25) that stabilization occurs for modes with $k_z = 0$ due to dust -neutral gas collisions only. The relative flow between dust and neutral gas tries to quench the instability, as one can see in case of no significant magnetic field the modes with wavenumber

$$k_y = \frac{k^2 g\left(\rho_{II} - \rho I\right)}{\left(\rho_I + \rho_{II}\right)\left(8v_{dn}^2 + \frac{1}{2}k_z^2 v_0^2\right)}$$

are stabilized. If the neutral gas is homogeneous i.e. $\rho_{nI} = \rho_{nII} = \rho_n$, the unstable modes grow in the dust fluid component and satisfy the criterion

$$\begin{gathered}\left(\rho_{dII} - \rho_{dI}\right)\left(\rho_{dI} + \rho_{dII} + 2\rho_n\right)\rho_n^3 g > \left(\rho_{dI} + \rho_{dII} + 2\rho_n\right)\rho_n^3 \frac{k_y k_z^2}{k^2}\frac{B_0^2}{\pi} + \\ \left[\frac{1}{4}\left(\rho_{dI} + \rho_{dII}\right)\left(\rho_{dI} + \rho_{dII} + 2\rho_n\right)^4 v_{dn}^2 + 2k_z^2 v_0^2\left(\rho_{dI} + \rho_{dII}\right)^2 \rho_n^3\right]\frac{k_y}{k^2}\end{gathered} \tag{27}$$

In the absence of relative velocity v_0 the instability condition reduces to

$$\begin{gathered}\left(\rho_{dII} - \rho_{dI}\right)\left(\rho_{dI} + \rho_{dII} + 2\rho_n\right)\rho_n^3 g > \left(\rho_{dI} + \rho_{dII} + 2\rho_n\right)\rho_n^3 \frac{k_y k_z^2}{k^2}\frac{B_0^2}{\pi} + \\ \left[\frac{1}{4}\left(\rho_{dI} + \rho_{dII}\right)\left(\rho_{dI} + \rho_{dII} + 2\rho_n\right)^4 v_{dn}^2\right]\frac{k_y}{k^2}\end{gathered} \tag{28}$$

If we simplify it, we obtain a similar condition to that condition (16) obtained by Birk [12] except a slight change in the constant coefficient of ρ_{dI}^5 and ρ_{dII}^5. If we consider homogeneous dust component i.e. $\rho_{dI} = \rho_{dII} = \rho_d$ and inhomogeneous neutral gas, the unstable growing modes satisfy the following condition

$$\begin{gathered}\left[\left(\rho_{nII}^2 - \rho_{nI}^2\right)\left(\rho_{nI} + \rho_{nII} + 2\rho_d\right)\right]\rho_d^2 g > \frac{1}{4}\left[\left(\rho_{nI} + \rho_{nII}\right)^3\left(\rho_{nI} + \rho_{nII} + 2\rho_d\right)\right]\times \\ \frac{k_y k_z^2}{k^2}\frac{B_0^2}{\pi} + \left[\left(\rho_{nI} + \rho_{nII} + 2\rho_d\right)^4 v_{dn}^2 + 2k_z^2 v_0^2\left(\rho_{nI} + \rho_{nII}\right)^3 \rho_d\right]\rho_d \frac{k_y}{k^2}\end{gathered} \tag{29}$$

In the absence of relative velocity v_0 this criterion reduces to

$$\begin{gathered}\left[\left(\rho_{nII}^2 - \rho_{nI}^2\right)\left(\rho_{nI} + \rho_{nII} + 2\rho_d\right)\right]\rho_d^2 g > \frac{1}{4}\left[\left(\rho_{nI} + \rho_{nII}\right)^3\left(\rho_{nI} + \rho_{nII} + 2\rho_d\right)\right]\times \\ \frac{k_y k_z^2}{k^2}\frac{B_0^2}{\pi} + \left[\left(\rho_{nI} + \rho_{nII} + 2\rho_d\right)^4 v_{dn}^2\right]\rho_d \frac{k_y}{k^2}v_{dn}^2\end{gathered} \tag{30}$$

which is similar to condition (18) obtained by Birk [12].

IV. CONCLUSIONS

In this paper, we have studied the effect of flow on fluid dynamical instabilities in magnetized partially ionized dense dusty quasineutral plasma with dynamics governed by dust and neutral gas components. The electron dynamics is not considered as electrons have no significant influence on the overall behaviour of dusty plasma. The instability conditions have been derived for three different cases. Thus, the onset criteria for unstable modes obtained by Birk [12] have been modified in the presence of relative flow in dust and neutral gas. The relative flow between dust and neutral gas has stabilizing effect on the system. The results might be useful in many situations of astrophysical magnetized dusty plasmas namely comets and circumstellar dusty disks e.g. T-Tauri stars.

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