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The Theory of Spontaneous Emission and Manifestation of Fermi's Golden Rule

J. Saikia[1*], B. Goswami[2] and G.D. Baruah[3]

[1] Department of Physics, J.B. College (Autonomous), Jorhat, Assam, India
[2] Salt Brook Academy, Dibrugarh, Assam, India
[3]Centre for Laser & Optical Science, New Uchamati, Doom Dooma, Assam, India

**Corresponding authors' E-mail: jibonsaikiajbc@gmail.com,Mobile +919435351338*

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Abstract - In the present work we have worked out a vector model for spontaneous emission based on the principle of moving vectors. This simplified approach reflects the work of Weisskopf and Wigner. Normalized probability of a transition to an upper level versus detuning has been plotted. Further we have shown that the areas which have been generated at phasor diagrams in different values of decay constants for moving vectors are directly proportional to time. This is a manifestation of Fermi's Golden rule. Fermi's Golden Rule in quantum mechanics is well known. But it is for the first time that we have worked out the concept of moving vectors using the transition probabilities and the concept of phasor diagrams to represent the Fermi's Golden rule.

Keywords: *Spontaneous emission, detuning, Moving Vector, Golden Rule.*

I. INTRODUCTION

The theory of spontaneous emission was given by Weisskopf and Wigner [1] several decades ago. The mechanism of spontaneous emission can be well understood from quantum theory of radiations [2]. It is an isotropic perturbation always present and attributed in connection with the quantum theory of radiation to all pervading zero point fluctuation of the electromagnetic field. The radiation affects the atoms: the zero point fluctuation deexcites them resulting in the re-radiation of light. An interesting consequence of the quantization of the radiation field is the fluctuation associated with the zero-point energy or the so-called vacuum fluctuation. These fluctuations have no classical analogy and are responsible for many interesting phenomena in quantum optics. In the usual picture of atom field interaction it can be shown that an atom in the upper level can make transition back and forth to the lower state in time even in the absence of an applied field. However, it is an excited state decays to the ground state with a characteristic life time but it does not make back and forth transitions. For a proper account of atomic decay a continuum modes corresponding to a quantized cavity, which is infinite in extents needs to be included. In Weisskopf-Wigner approximation the equation of motion for the probability amplitude is given by

$$\dot{C}_a = -\frac{\gamma}{2} C_a(t) \qquad (1)$$

where γ is the decay constant. A solution for Equation (1) given by

$$P_a \equiv |C_a(t)|^2 = \exp(-\gamma t) \qquad (2)$$

that is, an atom in the excited state $|a>$ in vacuum decays exponentially in time with the life time

$$\tau = \frac{1}{\gamma}$$

In the present work we report a vector model of spontaneous emission which actually reflects the works of Weisskopf and Wigner (1). In our earlier work we had indicated that the vector model worked out by us has lot of practical applications [3, 4]. In the present work we show for the first time that the areas generated from the phasor diagrams are proportional to time, a fact which is nothing but the manifestation of Fermi-Golden rule in quantum mechanics. The paper is organized as follows. In section II we introduced the concept of moving vectors using the formula for transition probability of absorption. How different phasor diagrams are constructed using the concept of moving vectors are illustrated here. In section III the manifestation of Fermi's Golden Rule in transition probabilities are shown. Section IV is a short discussion of the salient points of the findings of our work. Section V is concluding remark of the present work.

II. MOVING VECTORS

The energy level diagram for a two-level atom indicating decay γ_a and γ_b is shown in Fig.1.

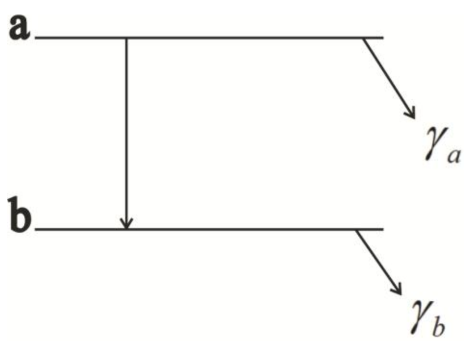


Fig.1: Energy Level diagram for two level atom with decay rates γ_a and γ_b

It is worthwhile to note here that the Weisskopf-Wigner theory of spontaneous emission justifies the inclusion of the phenomenological decay rates γ_a and γ_b in the Schrödinger equation for the transition probability amplitude. The probability of stimulated absorption including decay is given by

$$|C_a(t)|^2 = \left(\frac{\wp E_o}{2\hbar}\right)^2 \exp(-\gamma_b t)\frac{\sin^2\frac{(\omega-\upsilon)t}{2}}{\left(\frac{\omega-\upsilon}{2}\right)^2} \quad (3)$$

where $\wp$ is the electric dipole matrix element , ω is the atomic line centre frequency in laser media, υ is the laser (optical) oscillation frequency in radiation per second , not Hertz. The term $\omega-\upsilon$ is called detuning. The transition probabilities for arbitrary values of time may be plotted for different values of detuning. Thus using Eqn. (3) the graphs may be worked out which represents the evaluation of the population in the upper state for various values of detuning at 90, 100, 110, 120 and 130 MHz. This is shown in Fig.2.

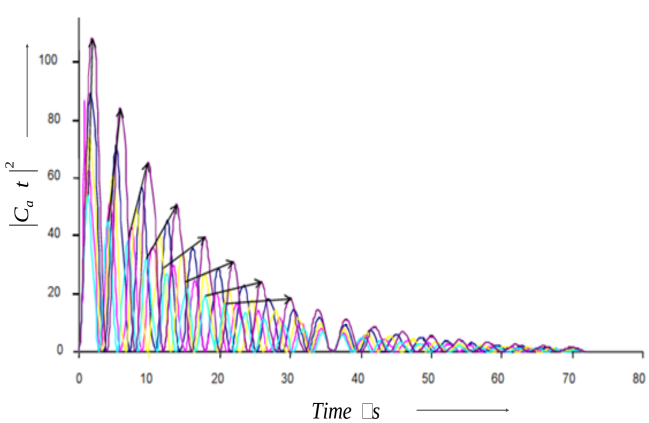


Fig2: Transition probabilities for five different values of detuning and γ =.005

As may be inferred from Fig.2 we may join the entire five maximum in a group of detuning (90, 100, 110, 120 and 130 MHz) as arrows and a number of arrows can be drawn in this way. We identify and define them as vectors, and call them moving vectors or they evolve in time. These vectors can be used to represent transition probabilities for change of population in the upper state. The zero transition probability occurs at arbitrary time of $6^2 n(n=1,2...)$ and it indicates some sort of collapse of wave function. It is worthwhile to note here that in the absence of the decay process, the semi-classical theory predicts Rabi oscillations for atomic inversion, whereas the quantum theory predicts certain collapse and revival phenomena due to the quantum aspect of the field. The Rabi model (5) is so named because of its original setting in nuclear magnetic resonance as studied by Rabi long ago. The collapse of Rabi oscillation was noted fairly early in the study of Rabi [6]. Several years later it was found that Rabi oscillation start to revive [7], although not completely. At longer times one finds a sequence of collapses and revivals, the revival becoming less distinct as time increases. These collapses and revivals behaviour of Rabi oscillations in the fully quantized model is strikingly different than in the semi-classical case. In our case we have seen that the collapses and revivals of transition probabilities is similar to the quantum model which is known as Jaymes-Cummings model [8]. In this connection we would like to indicate that the moving vectors can be represented by the equation

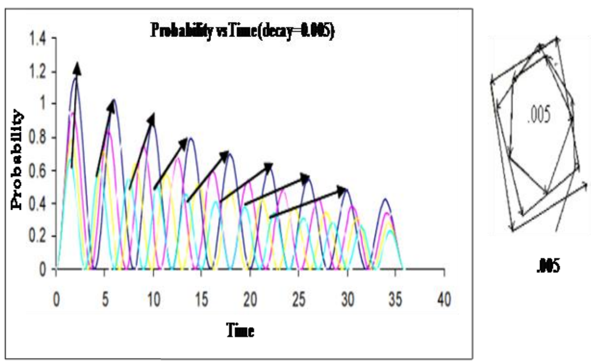
$$\frac{dk_n}{dt} = \Omega_n \times k_n \quad (4)$$

where, Ω_n is the driving field vector and k_n is the amplitude vector.

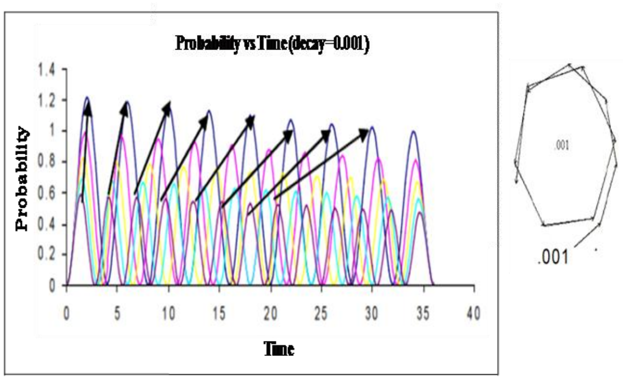
We have worked out different phasor diagrams with different decay constants and they are shown in Fig.3 [(a),(b),(c),(d),(e),(*f*)]. It is worthwhile to explain how we have constructed these phasor diagrams. To do this let us first consider the first vector which we have generated for a particular value of γ . At the arrow head of this vector we place the second vector at an angle determined by the orientation of the second vector with respect to the first. Similarly the third vector will be placed on the arrow head of the second vector and so on. We note here that the vectors in the phasor diagram first closes up and uncoil again and expands. It is also observed that when decay is small the expansion is rapid. This is the general nature of all the phasor diagrams we have constructed with different as shown in Fig 3 (a,b,c,d.e,*f*). It is of our immediate interest to discuss the nature of the areas enclosed by the moving vectors. But before that we find it worthwhile to discuss the topic of Fermi-Golden rule in quantum mechanics which has a direct application in our present work of phasor diagram.

III. GOLDEN RULE OF FERMI

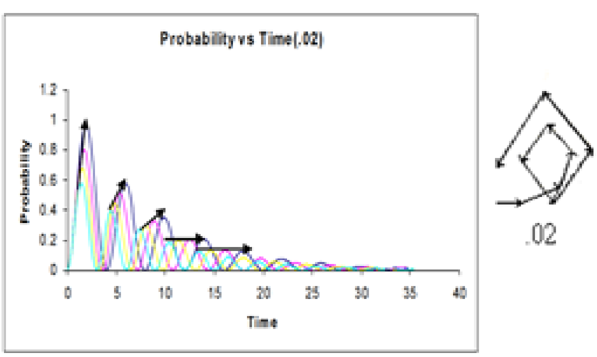
It is worthwhile to plot $\left|C_a(t)\right|^2$ versus detuning to get some physical insight of the problem. This is shown in Fig 4. From this figure it is seen that the probability of being in the upper state decreases rapidly as the detuning in increased. Furthermore the probability of being in the upper state decreases rapidly as the detuning in increased. The approximate width of the central peak is inversely proportional to t and the height is proportional to t^2. This gives an area under the curve proportional to t. This fact is the manifestation of the so-called golden rule of quantum mechanics, for its useful application.



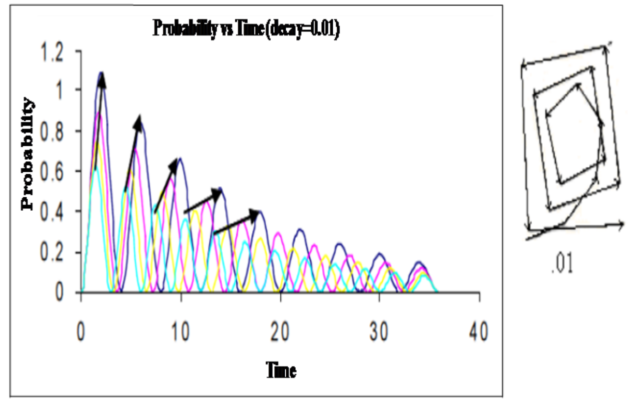
(a)



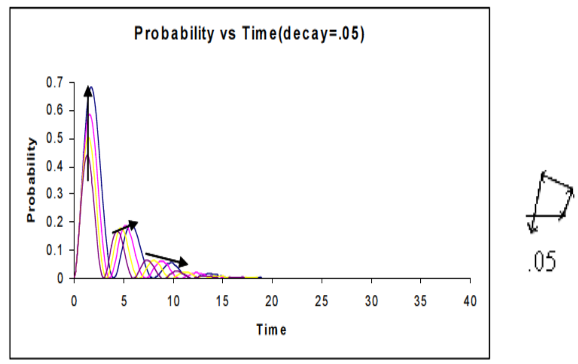
(b)



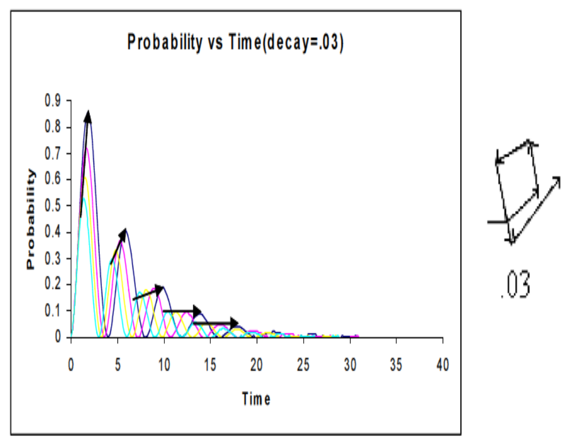
(c)



(d)



(e)



(f)

Fig.3 [(a), (b), (c), (d), (e), (f)]: Moving vectors for the values of decay constant $\gamma = .001, .005, .01, .02, .03$ and .05 and the corresponding phasor diagrams.This rule is based on the idea that the detuning, $(\omega - \upsilon)$, has a range of values due to a field with continuous spectrum, or to a spread in energy levels themselves, as applies to the photoionization of an atom in a detector.

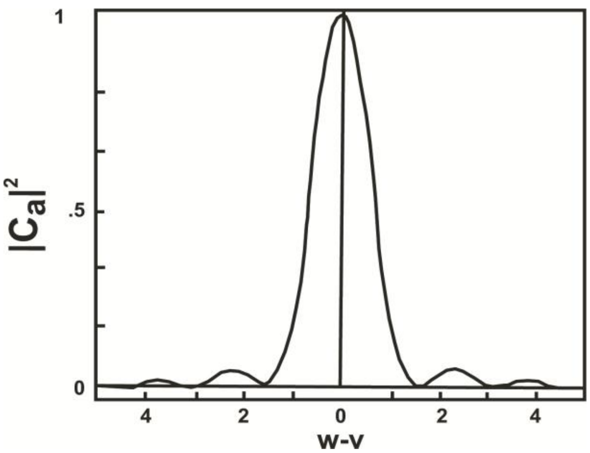


Fig.4: Normalized probability of a transition to an upper level versus detuning. (The area under the curve is approximately proportional to *t*.)

IV. RESULTS AND DISCUSSION

The affirmations in earlier section represent also what is observed in the areas enclosed in phasor diagrams. As may be inferred from Fig.3, the areas enclosed by the phasor diagrams depend on the value of the decay constants, γ which is nothing but inverse of life time, that is $\gamma = 1/\tau$. By plotting the value of areas against time we produce a plot which is shown in Fig 5.

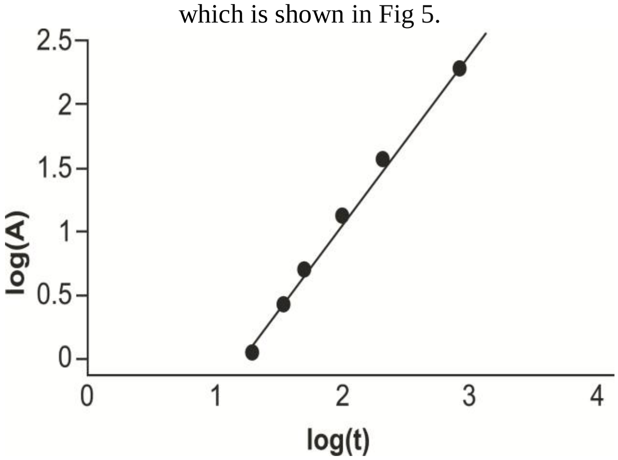


Fig.5: Decay versus Area.

As may be seen from Fig.5, the areas generated in phasor diagrams are proportional to time. Thus this is a manifestation of Fermi Golden Rule in quantum mechanics. It is worthy of remark that the concept of vectors to explain the equation of motion of Galileo is well known concept for all. As for example the interesting feature of the velocity time graph for any moving object is that the area under $\upsilon - t$ graph equals displacement of the object over a given time concept of area is used to represent vectors. As for example in definition of flux in electrostatics and magnetism the area is represented as vector. This is for the first time that we have worked out the concept of moving vectors using the transition probabilities and the concept of phasor diagrams to represent the Fermi's Golden rule.

V. CONCLUSION

In the present work we have worked out a vector model based on the formula representing the probability of stimulated absorption.

$$\left|C_a(t)\right|^2 = \left(\frac{\wp E_o}{2\hbar}\right)^2 \exp(-\gamma_b t) \frac{\sin^2 \frac{(\omega-\upsilon)t}{2}}{\left(\frac{\omega-\upsilon}{2}\right)^2}$$

Our simplified approach only reflects the work of Wigner and Weisskopf's theory of spontaneous emission. The area enclosed by the phasor diagrams which we have constructed also manifest Fermi's Golden rule. It is reasonable to believe that the moving vector model will have practical application in various fields.

REFERENCES

[1] V.F. Weisskopf and E. Wigner, Z. Physik 39, 54 (1930).
[2] M. Sargent III, M.O. Scully and W.E. Lamb Jr., Laser Physics (Addison-Wesley Publishing Company, Reading, Massachusetts) 1974.
[3] F.B. Hazarika, S.N. Rai and G.D. Baruah, OPTIK, Int. J. Light. Electron. Opt. 122, 85 (2011). [4] F.B. Hazarika, B. Goswami, H. Konwar and G.D. Baruah, Asian J. Spectroscopy, Special issue, **291-294,** (2012).
[5] I.I. Rabi, Phys. Rev. 51, 652 (1937).
[6] F.W. Cummings, Phys, Rev. 149, 104 (1963).
[7] J.H. Eberley, N.B. Narozhny and J.J. Sanchez-Mondragon, Phys, Rev. Letts. 44, 1323 (1980).
[8] E. Jaynes and F.W. Cummings. Prr. IEEE, 51, 89 (1963).

