International Journal of Scientific Research in Physics and Applied Sciences

Research Paper

Vol.7, Issue.5, pp.07-10, October (2019)

E-ISSN: 2348-3423

Maxwell's Generalisation: Displacement Current

S.K. Parida

Dept. of Physics, ITER, Siksha 'O' Anusandhan Deemed to be University, Bhubaneswar 751030, India

Corresponding Author: santoshparida@soa.ac.in

Available online at: www.isroset.org

Received: 10/Oct/2019, Accepted: 20/Oct/2019, Online: 31/Oct/2019

Abstract- The enlightenment of the article is to introduce the basic concepts and terminology for the understanding of the electromagnetic induction i.e. the relation of electricity and magnetism. A special attention has been given to the concept of displacement current introduced by James Clerk Maxwell to explain the equation of continuity of charges. The changing electric field is the source of the magnetic field and vice versa. During the discussion of the topic, we make an attempt to understand some terminology, the typical value of principles & all boxed equations.

Keywords- displacement current, electromagnetic induction, electrodynamics, magnetic monopole

I. INTRODUCTION

Displacement current is one of the most interesting controversial topics that have its standpoints in classical electrodynamics [1-4]. Many research publications and books on the topic of electrodynamics provide the insight history of the concept of the Maxwell's displacement current [5-9]. For understanding the concept of the displacement current, we must answer some general questions related to the meaning, real or virtual current, physical interpretation, relation to steady and unsteady current etc. In 1865, James Clerk Maxwell [10] delivered a seminar talk on the keyword of the very famous four equations of electrodynamics universally known as Maxwell's equations and concludes the topic with the connection of electromagnetic waves that travel with speed $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ in the vacuum, where ε_0 is the permittivity in free space and μ_0 is permeability in free space. The Maxwell's equations in the free space are

i. $\nabla . \vec{E} = \frac{\rho}{\varepsilon_0}$ (Gauss's law in Electrostatic) (Charge is the source of an electric field)

ii. $\nabla . \vec{B} = 0$ (Gauss's law in magnetism) (Non-existence of magnetic monopole) or magnetic field lines always form closed loop i.e they have no starting and ending point.

iii. $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday's law of electromagnetic induction)
(Rate of change of magnetic field is the source of an electric field)

iv. $\nabla \times \vec{B} = \mu_0 \vec{J}$ (Ampere's Circuital law)
(Current is the source of a magnetic field)

The first three equations seem to be consistency with old mathematical operators' divergence, curl but eq.(iv) gets trouble. Taking divergence over fourth equation, it can be written as $\nabla . (\nabla \times \vec{B}) = \mu_0 (\nabla . \vec{J})$. The left side of the above equation is zero because of divergence over curl, but the right-hand side of the equation (iv) is not zero. For steady currents, the divergence of J is zero, but when we go beyond magnetostatics, the Ampere's law failed. Therefore, one can check the Ampere's law for non-steady currents. Let us consider a conductor carrying steady current is placed horizontally and bring a magnetic compass needle near to it. The deflection of needle confirms the presence of the magnetic field shown in fig.1. According to Ampere's Circuital Law (ACL), there must be a current due to which actual movement of charged particles take place called as conduction current; is the source of a magnetic field.

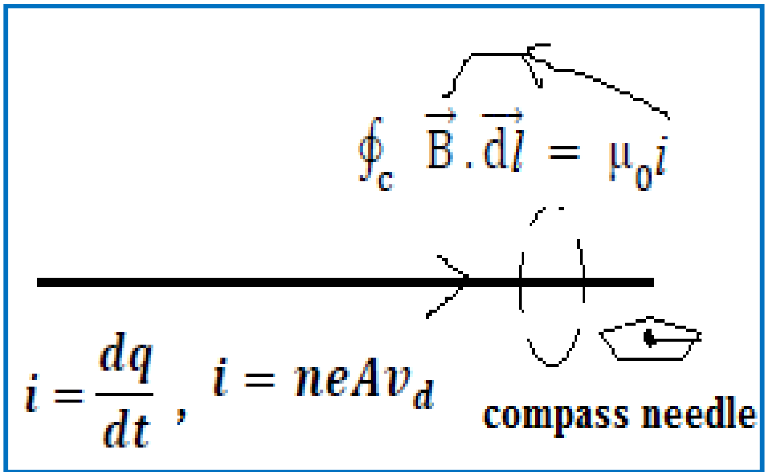

Figure 1. Magnetic field due to a straight conductor carrying current

But for non-steady currents (arrange of parallel plate capacitor) in which ACL seems to fail. This difficulty is overcome by Maxwell, introducing the concept of displacement current. Maxwell explained the Ampere's law using the law of symmetric saying that if a varying magnetic field is the source of an electric field (Faraday's law), why not changing electric field would be the source of the magnetic field [11]. Then, the current associated with the time rate of change of electric field is called displacement current. Mathematically, $\boxed{\mathbf{i_d = \varepsilon_0 A \frac{\partial E}{\partial t}}}$ where i_d = displacement current, A = area of the each parallel plate, E = electric field applied & t = time taken. So, in summary we may say that

Varying magnetic field is the source of electric field; changing electric field is also the source of magnetic field.

II. RESULTS AND DISCUSSION

Let us consider a coil of area A is placed inside a parallel plate capacitor as shown in fig.2. Now it is connected with an external ac source to charge the capacitor. The compass needle deflection shows the presence of conduction current outside of the capacitor. But the deflection of the needle in between the parallel plates cannot understand [12]. The electric field set up between the parallel plate changes with time and produces a magnetic field in absence of conduction current. According to Ampere's circuital law, $\oint_c \vec{B}.\vec{dl} = \mu_0 i_{net}$, the existing current in the right side is due to time rate of change of magnetic field [13]. To explain the induced current (other than conduction current), Maxwell has modified the Ampere's circuital law by introducing the concept of displacement current. So the time rate of change of electric field between the parallel plates produces a current called as displacement current (i_d).

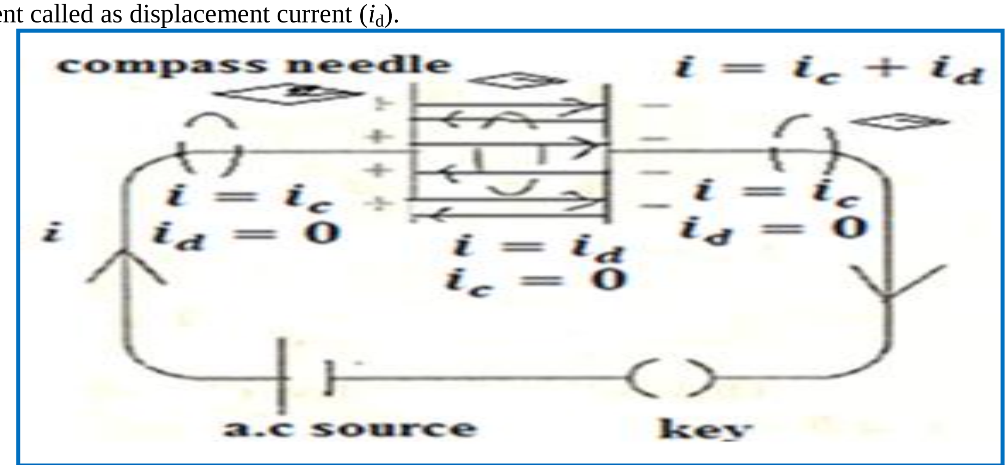

Figure 2 Arrangement of parallel plate capacitor for existence of displacement current

The modified Ampere's circuital law can be derived from the point that the electric field between two parallel plates of area *A* and charge *q* is given by

$E = \frac{q}{\varepsilon_0 A}$ ($\varepsilon_0 =$ permittivity in free space)

Taking derivative of time *t* on either side, we get

$$\Rightarrow \frac{\partial E}{\partial t} = \frac{1}{\varepsilon_{0A}} \frac{\partial q}{\partial t}$$

$$\Rightarrow \frac{\partial E}{\partial t} = \frac{1}{\varepsilon_{0A}} i_d$$

$$\Rightarrow i_d = \varepsilon_0 A \frac{\partial E}{\partial t}$$

$$\boxed{i_d = \varepsilon_0 A \frac{\partial E}{\partial t}}$$ (for vacuum) **Working formula**

In terms of capacitance C of the given capacitor, the expression of displacement current can be derive as

We know $q = CV$

Taking derivative of time *t* on either side, we get

$$\Rightarrow \frac{\partial q}{\partial t} = \frac{\partial}{\partial t} \mathrm{CV}$$

$$\Rightarrow I_d = C \frac{\partial V}{\partial t}$$

$$\boxed{i_d = C \frac{\partial V}{\partial t}}$$ **working formula**

The expression for displacement current in term of current density

We know, $i_d = \varepsilon_0 A \frac{\partial E}{\partial t}$

$$\frac{i_d}{A} = \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\boxed{\overrightarrow{J_d} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$$ **working formula**

Where J_d is the displacement current density.

Maxwell has modified the Ampere's circuital law by adding the concept of displacement current so-called as Modified Ampere's Circuital Law/ Maxwell-Ampere's Law. The right-hand side of the Ampere's law $i_{net} = i_c + i_d$, therefore the modified Ampere's law can be written in the new form as

$$\oint_c \vec{B} . \vec{dl} = \mu_0 i_{net}$$

$$\Rightarrow \oint_c \vec{B} . \vec{dl} = \mu_0 (i_c + i_d)$$

$$\Rightarrow \oint_c \vec{B} . \vec{dl} = \mu_0 \left[i_c + \varepsilon_0 A \frac{\partial E}{\partial t} \right]$$

$$\Rightarrow \oint_c \vec{B} . \vec{dl} = \mu_0 \left[i_c + \varepsilon_0 \frac{\partial (EA)}{\partial t} \right]$$

$$\boxed{\oint_c \vec{B} . \vec{dl} = \mu_0 \left[i_c + \varepsilon_0 \frac{\partial \varphi_E}{\partial t} \right]}$$

This is the expression of Modified Ampere's Circuital law (MACL). So; on the conclusion the presence of displacement current satisfied equation of continuity of charge.

III. CONCLUSION

An attempt has been made to explain the concept of Maxwell's displacement current. The current associated with a rate of change of electric field with time provides a current called displacement current. It does not obey Ohm's law. It depends upon medium between parallel plates. Without the existence of displacement current, the equation of charge continuity failed. So, it may conclude that changing the electric field is the source of the magnetic field that shown inside the capacitor. So, electric and magnetic fields are correlated to each other and form the self sustaining electromagnetic waves which move with speed of light.

ACKNOWLEDGMENT

I am thankful to Department of Physics, ITER, Siksha „O‟ Anusandhan (Deemed to be University), Bhubaneswar, for giving me support.

REFERENCES

[1]. F. W. Warburton, "*Displacement Current, A Useless Concept*", American Journal of Physics, Vol. 22, pp. 299-305, 1954
[2]. J. W. Arthur, "*An Elementary View of Maxwell's Displacement Current*", IEEE Antennas and Propagation, Vol. 51, No. 6, pp. 58-68, 2009
[3]. S. K. Parida, *Polymer Nanocomposites and Applications: A Brief Review*, International Journal of Scientific in Research in Physics and Applied Sciences, Vol.6, Issue 3, pp.75-78, 2018
[4]. W. G. V. Rosser, "*Displacement Current and Maxwell's Equations*", American Journal of Physics, Vol. 43, No.6, pp. 502-505, 1975
[5]. J. Kumari, R. S Pandey, G Varughese and K. M Singh, "*Oblique Electromagnetic Ion Cyclotron Instability with A.C. Electric field for Loss-cone Distribution Function*", International Journal of Scientific in Research in Physics and Applied Sciences, Vol.5, Issue 4, pp.1-12, 2017
[6]. W. G. V. Rosser, "*Does the Displacement Current in Empty Space Produce a Magnetic Field*", American Journal of Physics, Vol. 44, pp. 1221-1223, 1976.
[7]. J. Roche, "*The Present Status of Displacement Current*", European Journal of Physics, Vol. 19, pp. 155-166, 1998.
[8]. J. D. Jackson, "*Maxwell's Displacement Current Revisited*", European Journal of Physics, Vol. 20, pp. 495-499, 1999.
[9]. G. B. Walker, "*The Illusory Currents of Classical Electromagnetism*", Physics Essays, Vol. 20, No. 3, pp. 473-477, 2007.
[10]. J. C. Maxwell, "*A Dynamical Theory of the Electromagnetic Field*", Transactions of the Royal Society, Vol. 155, pp. 459-512, 1865.
[11]. John David Jackson, "*Classical Electrodynamics*", 3rd ed., Wiley India Pvt. Limited, New Delhi, pp-666, 1999.
[12]. Leigh Page, Norman IIsley Adams, "*Electrodynamics*", Chapman and Hall, Chapter-7, 1940.
[13]. David J Griffiths, "*Introduction to Electrodynamics*", 3rd ed., Prentice-Hall Inc., New Jersy, Chapter-7, 1999.

Author Profile

Santosh Kumar Parida completed his PhD degree at Siksha 'O' Anusandhan Deemed to be University, Bhubaneswar, India. His teaching and research experience spans over more than one decades including B.Tech, M.Sc & Ph.D. His studies involve synthesis of nano-particles and nanocomposites, alloys system and mathematical modelling and simulation. He has contributed many research papers in national/international journals of repute.