Quark Gluon Plasma (QGP) Evolution under Loop Corrections

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Abstract-We review free energy evolution of QGP (Quark-gluon plasma) under zero-loop, one loop and two loop corrections in the mean field potential. The free energies of QGP under the comparison of zero-loop and loop corrections of the interacting potential among the quarks, anti-quarks and gluons are shown. We observe that the formation of stable QGP droplet is dependent on the loop corrections with the different parametrization values of fluid. With the increase in the parametrization value, stability of droplet formation increases with smaller size of droplet. This indicates that the formation of QGP droplet can be signified more importantly by the parametrization value like the Reynold number in fluid dynamics. It means that there may be different phenomenological parameter to define the stable QGP droplet when QGP fluid is studied under loop corrections.

Keywords- Quark-Gluon Plasma, Quark-Hadron Phase Transition, Loop Correction

I. INTRODUCTION

The theory of strong interactions states about the prediction of quark-hadron phase transition under the condition of extreme high nuclear density and very high temperature. In the transition phenomena, matter consists of free quarks and gluons called quark-gluon plasma (QGP) turns into a matter of confined phase of bound quarks of hadrons [1–4]. The system is believed to exist for a short time and the study of this short span makes us complicated in search of the transition phenomena. It is believed that the beginning of early universe expansion, which is de-scribed by Big Bang theory, was very hot and subsequently it becomes cool down with the expansion of the universe. So phase transition is in due progress of universe's subsequent ex-pansion. One way to explain the phase transition is through a process of studying very high temperature system obtained in the Laboratory. Another process about the phase transition is through the process of studying very high nu-clear density matter. These very high nuclear density matter is normally obtained in the formation of compact objects, neutron star and bo-son stars [5–9]. These objects are believed to be found after the death of giant stars in fiery explosion called supernova explosion. For the investigation and search about the nature of the universe, several experimental facilities are set up around the globe like relativistic heavy-ion collision (RHIC) at BNL and large hadron collider (LHC) at CERN. These two experiments have examined about the creation and forma-tion of our universe by colliding head on head with very energetic ion-beams. These experiments have also claimed for the creation of mini universe called quark-gluon plasma (QGP) [10– 12]. So these two experiments give the information about matter at very high temperature. On other hand there are some experimental

facilities like FAIR at Darmstadt and NICA at Dubna, where the study have focused on dense baryonic matter and the baryonic matter at Nu-clotron (BM@N) experiments which extracted ion beams from modernized Nuclotron, will pro-vide the future information about the formation of QGP under the influence of compressed dense nuclear matter [13–18]. All the facilities avail-able so far are trying to detect the existence of the critical point in the phase structure, the early universe phase transition, formation of QGP and chromodynamics (QCD) phase structure at very high nuclear density. So, the investigation on quark-gluon plasma (QGP) through the Ultra Relativistic Heavy-Ion Collisions has become an exciting field in the current scenario of heavy ion collider physics. In this review article, we focus on the calculation of bulk thermodynamic properties of these matter at very high temperature in continuation of our earlier works of zero loop and one loop correction incorporating the two loop correction [19, 20]. It has been reported more stable droplets of QGP and the corresponding parametrization value used in the two loop has been largely affected in the droplet size formation. So the droplet evolution under two loop correction is more likely to predict the changes in the stability of droplet formation.

II. POTENTIALS OF LOOP CORRECTION:

The interacting potential among the quarks and anti-quarks through zero-loop is defined in the following as

$$V_{zero}(p) = \frac{8\pi}{p}\gamma\alpha_S(p)T^2 - \frac{m_o^2}{2p} \qquad (1)$$

the system. Then the potential which is obtained when one loop correction is introduced in

$$V_{one}(p) = V_{zero}\left[1 + \frac{\alpha_s(p)a_1}{4\pi}\right] \quad (2)$$

in which γ is the parametrization value which is taken in terms of quark and gluon parametrization factors. The value is different depending on zero, one and two loop corrections [21, 22]. In zero-loop case the value of quark and gluon parametrization
is $\gamma_q = 1/6$ and $\gamma_g = (6-8)\gamma_q$ for stable droplet for-mation whereas it is taken as $\gamma_q = 1/8$ and $\gamma_g = (8-10)\gamma_q$ for one loop correction. So we extend to look for the potential for the case of two loop.

$$V_{two}(p) = V_{zero}\left[1 + \frac{\alpha_s(p)a_1}{4\pi} + \frac{\alpha_s^2(p)a_2}{16\pi^2}\right] \quad (3)$$

The value of parametrization for the case of two loop correction to find the stable droplet for-mation is $\gamma_q = 1/14$ and $\gamma_g = (48-52)\gamma_q$. All these factors determine the stability of droplet as well as dynamics of QGP flow similar to the standard Reynold's number of liquid flow and these parameters support in forwarding the transformation to bound state of matter called hadrons. In addition to these parameters we need to de-fine another parameter known as correction loop parameter a_1 for one loop in the potential equa-tion, which is obtained through the interacting potential among the particles. So the coeffcient a_1 is one loop correction in their interactions and it is given as [23–25]:

$$a_1 = 2.5833 - 0.2778n_l \quad (4)$$

where n_l is the number of light quark ele-ments [26–29]. The value is defined for one loop correction and we extend our calculation into further interactions up to the factor of two loop correction. In the case of two loop correction the coefficient is obtained as:

$$a_2 = 28.5468 - 4.1471n_l + 0.0772n_l^2 \quad (5)$$

Similarly we have the different mass factors de-pending on the loop. The mass of the
corresponding one loop correction is defined as:

$$m_{one}^2(T) = 2\gamma^2 g^2(p)T^2[1 + g^2(p)a_1] \quad (6)$$

Whereas the mass of two loop correction is: $m_{two}^2(T) = m_{one}^2(T) + 2\gamma^2 g^6(p)T^2 a_2$ (7)

These thermal masses obtained after incorporation of one and two loop corrections play pivotal role to find the interacting potential. Really the interaction potentials of loop are probably created due to the thermal effective mass of quarks and anti-quarks. Now to obtain our aim we fur-ther look the grand canonical ensemble through these loop corrections.

III. GRAND CANONICAL ENSEMBLE AND FREE ENERGY

We evaluate the grand canonical ensemble through the loop correction in the interaction potential of quarks, anti-quarks by the exchange of the coloured particle called gluon. So the evolution obtained through the ensemble is de-fined through density of state of the system. The density of state is defined in such way that mean field potential through the loop correction is incorporated. The free energies of quarks, gluons and hadrons can be obtained through the thermodynamic canonical ensemble of the system. The general partition function of the system defined by many authors is given [30]

$$Z(T,\mu,N) = Tr\left\{\exp[-\beta(\hat{H}-\mu\hat{N})]\right\} \quad (8)$$

where, μ is chemical potential of the system, N is quark number and $\beta = \frac{1}{T}$. Using this partition function, we correlate the free energy through the density of states incorporating the loop cor-rection factor, and it is calculated by [31–33]:

$$F_i = T\ln Z(T,\mu,N) \quad (9)$$

$$F_i = \eta T g_i \int \rho_i(p)p^2 \ln[1 + \eta\exp\{-(\sqrt{m_i^2+p^2}-\mu)/T\}dp \quad (10)$$

where ρ_i is the corresponding den-sity of states of loop, say i =(zero-loop, one loop and two loop), $\eta = -1$ for the bosonic particle and η = + for fermionic particles. In the formalism of these free energies we use the density of states which is derived through Thomas and Fermi model in which the corresponding unloop, one and two loop correction are incorporated in the interacting mean field potential. g_i is degree of freedom for quarks and hadronic particles. The value of this degree of freedom is given as 6 for quarks and 8 for gluons. It is not a number in the case of hadronic particles which is defined as $g_i = dv/(2\pi^2)$ where d is number factor like as degree of freedom of quark and gluon depending on the particular hadronic particle and $v = \frac{4/3}\pi r^3$ represents as the volume of the hadron droplet. However, the density of states ρ_i for hadronic particles is unity with its momentum factor p^2 and it is defined for the case of quarks and gluon in a way that mean field potential through the loop correction is incorporated with unit momentum factor and corresponding density of states [34]. The density of states for quarks and gluon for the corresponding loops are defined below and when the loop is incorporated, the corresponding density of state is applied in the calculation of their free energies using the corresponding mean field potential.

$$\rho_i(p) = v/3\pi^2(dv_{conf}^3(p)/dp) \quad (11)$$
Or
$$\rho_{zero}(p) = v/\pi^2(\gamma^9 T^6/8)g^6(p)A \quad (12)$$

Where,

$$A = (1/p^2)[(1/p^2) + 2/(p^2 + \Lambda^2)\ln(1 + p^2/\Lambda^2)] \quad (13)$$

whereas in the case of one loop correction the density of state is obtained as:

$$\rho_{one}(p) = \nu/\pi^2(\gamma^9 T^6/8)g^6(p)B \quad (14)$$

$$B = \{1 + \alpha_s(p)a_1/T_l\}^2[(1 + \alpha_s(p)a_1/\pi)/p^4 \quad (15)$$
$$+ 2(1 + 2\alpha_s(p)a_1/\pi)/p^2(p^2 + \Lambda^2)\ln(1 + p^2/\Lambda^2)]$$

Similarly, we obtain the density of state for two loop system as follows:

$$\rho_{two}(p) = (\nu/\pi^2)(\gamma^9 T^6/8)g^6(p)C \quad (16)$$

Where

$$C = [1 + (\alpha_s(p)a_1)/\pi + (\alpha_s^2(p)a_2)/\pi^2]^2$$
$$\times [(1 + 2\alpha_s(p)a_1/\pi + \alpha_s^2(p)a_2/\pi^2)/p^4 \quad (17)$$
$$+ 2(1 + 2\alpha_s(p)a_1/\pi + 3\alpha_s^2(p)a_2/\pi^2)$$
$$\div p^2(p^2 + \Lambda^2)\ln(1 + p^2/\Lambda^2)]$$

It implies that due to the correction factor, the density of states is perturbed by small factor which can be seen in the figure of potential vs momentum. In the expression, the parameter Λ is considered in the scale of QCD as 0.15 GeV. So we can set up the free energy of the system by finding the energies of quarks, anti-quarks, gluons, all the light and medium light hadrons. The integral is evaluated from the least value of momentum approximately tending to zero. Taking and considering all the massive hadrons now the total free energy is calculated by adding the inter-facial energy of the fireball [35, 36].

$$F_{total} = \sum_i F_i + (\gamma TR^2/4)\int p^2 \delta(p - T)dp \quad (18)$$

in the first term of the total free energy the summation in which i stands for u, d, s quarks and all the hadronic particles with gluon whereas in second term, it is inter-facial energy which re-place the role of bag energy of MIT bag model in which Bag energy was introduced in the scale of $B^{1/4} = T_c$. Taking the inter-facial energy in place of MIT Bag energy, it can reduce the draw-back produced by Bag energy to the maximum effects in comparison to MIT model calculation and this inter-facial energy is dependent on tem-perature and some parametric factor. So in the inter-facial energy, R is size of QGP droplet with the parametrization factor.
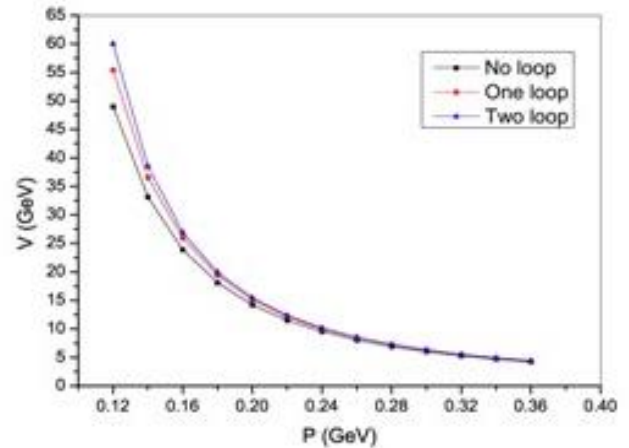

Fig. 1: Potential vs. Momentum with and without loop.

IV. RESULTS

The analytical calculations of free energy of QGP-hadron fireball evolution without and with one and two loop correction factors in the in-teracting mean-field potential are performed by computing the variation of the interacting potential and momentum. The potential function is
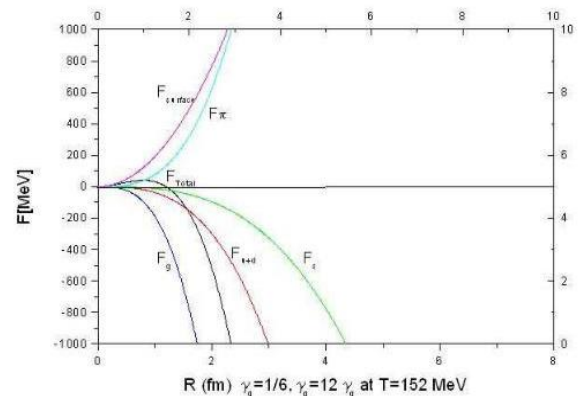

Fig. 2: Free energy vs. R at T = 152 MeV for zero-loop for contribution particles.
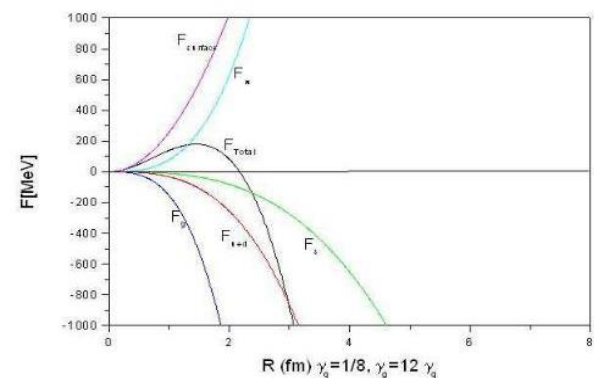

Fig. 3: Free energy vs. R at T = 152 MeV for one loop for contribution particles.

slightly perturbed from the unloop factor. These characteristic feature is shown in Fig.1. By the loop correction it is slightly increased at the lower momentum region and with increasing momentum, the perturbative contribution is negligible indicating that the perturbations of the one and two loop correction are very small in the high momentum transfer. Then we look for-ward the free energy change with droplet size of the different contributed particles for the un-loop potential at a particular temperature say T= 152 MeV as ad-hoc assumption. At this particular temperature T = 152 MeV, we again look the change in behaviour of free energy of the constituted particles for one loop potential as considered in case of unloop potential and we obtain the similar behaviour with a slight difference in free energy amplitude. It implies that there is similar phenomenon of zero-loop and loop In
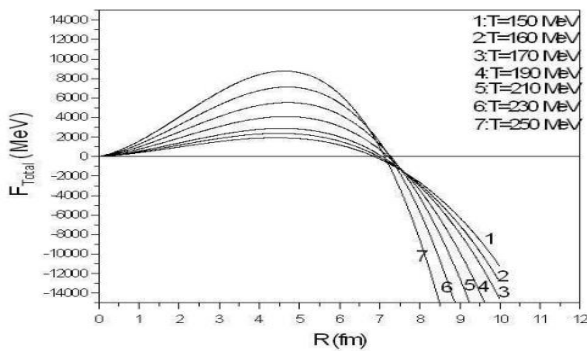

Fig. 7: Free Energy vs. R (fm) at $\gamma_Q = 1/8$, $\gamma_G = 10\gamma_Q$ for one loop correction

the free energy graph with different amplitudes at any particular temperature.

Now in Fig.4, we show free energy evolution for zero-loop potential for different temperatures at parametrization $\gamma_q = 1/6$ and $\gamma_g = 6\gamma_q$. This particular parametrization $\gamma_q = 1/6$ and $\gamma_g = 6\gamma_q$ which we chosed is due to the finding of the stable droplet. The value is obtained after ad-hoc search of stable droplet. So there is only one stable droplet formation for unloop potential. If we further look stable droplet by in-creasing the parameter beyond at $\gamma_q = 1/6$ and $\gamma_g \geq 6\gamma_q$, there is no more stable droplet at any other parametrization value which is represented by Fig.5. It means the stable droplet formation is obtained specially at parametrization value say $\gamma_q = 1/6$ and $\gamma_g = 6\gamma_q$ and other droplets are not exactly stable in unloop potential even though droplets be formed. Beyond $\gamma_g > 6\gamma_q$ there is droplet but no stability. It denotes that the parameter value behaves like a representa-tion to control in the behaviour of fluid dynam-ics in terms of its stable droplet formation. How-ever when the value of γ_q is not 1/6 and γ_g is not equal to six times of γ_q then we can have unsta-ble QGP formation and QGP fluid may be any sorts of unpredictable fluid dynamics. In Fig.6 and 7, there are free energy representations of one loop showing smaller droplet size with dif-ferent temperatures. It indicates that we have found similar stable droplets at the parametriza-tion values $\gamma_q = 1/8$ and $8\gamma_q \leq \gamma_g \leq 10\gamma_q$ in the case of additional oneloop potential.

In Fig.6 it shows changing slightly in the sta-

Fig. 4: Free Energy vs. R (fm) at $\gamma_Q = 1/6$, $\gamma_G = 6\gamma_Q$ for zero-loop correction
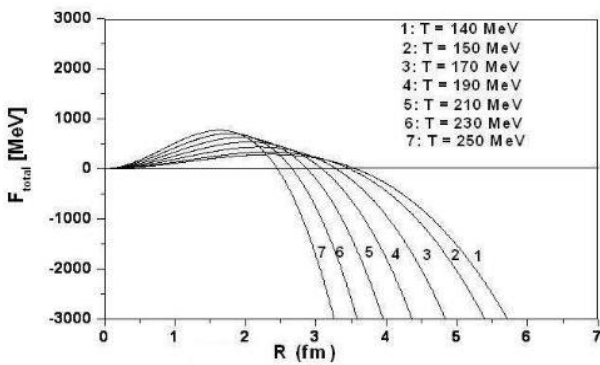

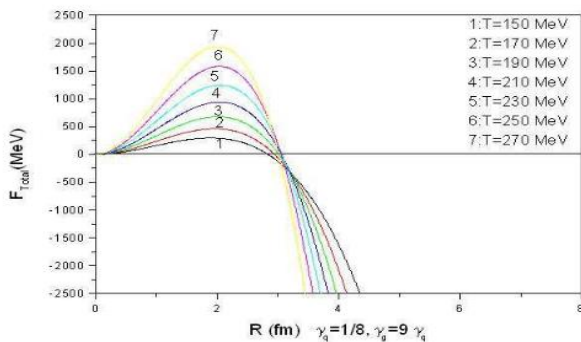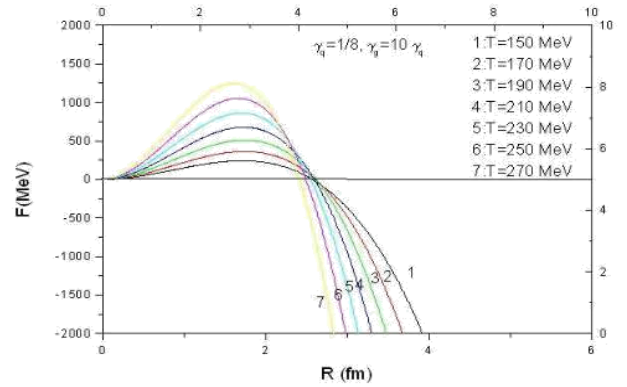Fig. 5: Free Energy vs. R (fm) at $\gamma_Q = 1/6$, $\gamma_G = 8\gamma_Q$ for zero-loop correction.

Fig. 6: Free Energy vs. R (fm) at $\gamma_Q = 1/8$, $\gamma_G = 9\gamma_Q$ for one loop correction
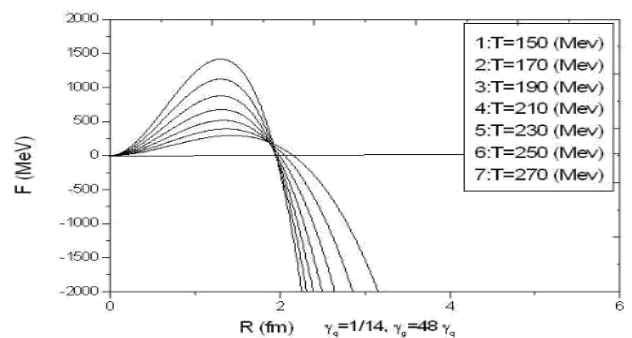

Fig. 8: Free Energy vs. R (fm) at $\gamma_Q = 1/14$, $\gamma_G = 48\gamma_Q$ for two loop correction.
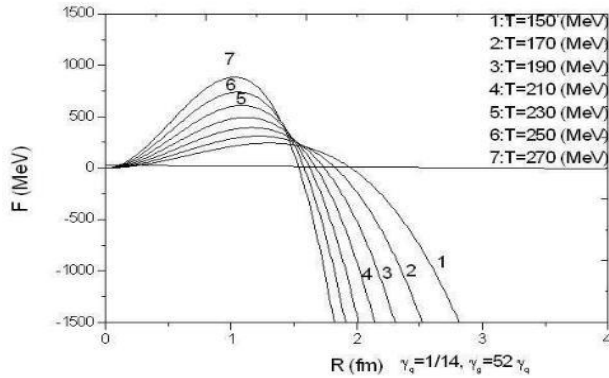
Fig. 9: Free Energy vs. R (fm) at $\gamma_Q = 1/14$, $\gamma_G = 52\gamma_Q$ for two loop correction.

ble droplet size from droplet of unloop potential. Similarly in Fig.7 stable droplet is available to obtain by changing the parametrization value to $\gamma_g \leq 10\gamma_q$. From these two presentations, there are almost around two stable droplets formations and these stability's are only found in the range of parametrisation $8\gamma_q \leq \gamma_g \leq 10\gamma_q$. In a similar way, we look phenomenologically about the evolution of free energy by adding twoloop correction in the potential. The behaviour of droplets are shown in Fig.8 and Fig.9. The behaviour of evolution of free energy with the increase of QGP drop size is much smaller in comparison to earlier droplets obtained by unloop and one loop correction in the potential. The addition of such two loop correction shows the forma-tion of stable droplet modified a lot and there is change in the parameter factor also. The stable droplet is still observed for different parameter values. The values are found to be $\gamma_q = 1/14$ and $48\gamma_q \leq \gamma_g \leq 52\gamma_q$. Only at these values we can observe the stable QGP droplet for two loop correction. Beyond these parametrization, QGP fluid remains like unstable behaviour of zero-loop type with having different in-oriental fluid dynamics and represent unstable droplets. These characteristic features are all described in the corresponding figures 8 and 9. With the in-corporation of two loop, the droplet size is much smaller indicating highly stable. The smaller the size, surface tension of drop is large and the liq-uid drops are tightly bounded so that the droplet is more stable. So parametrization factors for zero-loop, one loop and two loop play a very much important role in finding the stable droplet in the evolution of QGP droplet formation and it is really significant parameter for formation of QGP droplets.

V.   CONCLUSION

We can conclude from these results that due to the presence of loop corrections in the mean field potential, the stability is found to be much better in the case of two loop correction by the smaller size of droplet which is not comparable in terms of one loop correction and without loop correction where sizes of droplets are found to be bigger. So loop corrections in the potential can be studied as phenomenological model for describing the droplet formation of QGP taken as a dynamical parameter.

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