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Research Article

Analytical Analysis of Convection in Rotatory Walters B´ Viscoelastic Fluid in Presence of Magnetic Field

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Abstract — An attempt has been made to investigate the effects of uniform vertical magnetic field and uniform rotation on thermal convection in Walters B' viscoelastic fluid. Following the linearized stability theory, Boussinesq approximation and normal mode analysis, the dispersion relation is obtained. In the scenario of stationary convection, Walters B' viscoelastic fluid behaves like a Newtonian Fluid. Further, it is found that rotation has a stabilizing effect whereas the magnetic field has both stabilizing and destabilizing effects. In addition to this, it has been discovered that the system is reliable for $\frac{g\alpha\kappa}{\nu\beta} \leq (1-2\sigma_r F)\frac{27\pi^4}{4}$ and under the conditions $\frac{g\alpha\kappa}{\nu\beta} > (1-2\sigma_r F)\frac{27\pi^4}{4}$ and $0 \leq \sigma_r \leq \frac{1}{2F}$ the system goes into an unstable state. Overstability has also been looked at from the perspective of a scenario in which sufficient circumstances are met to rule out the possibility of the phenomenon occurring. It has been discovered that the rotation, magnetic field and viscoelasticity introduce oscillatory modes in the system which were non-existent in their absence.

Keywords — Thermal Convection; Walters B' Viscoelastic Fluid; Uniform Vertical Magnetic Field; Uniform Rotation; Linear Stability Theory; Normal Mode Analysis method

1. Introduction

A comprehensive account of thermal convection (Be'nard convection) in a fluid layer, in the absence and presence of rotation and magnetic field has been given [1]. The use of Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in its association with the external force. The approximation is well justified in the case of incompressible fluids. The influence of Rayleigh-number in turbulent and laminar region in parallel-plate vertical channels has been studied [2]. The influence of radiation on the unsteady free convection flow of a viscous incompressible fluid past a moving vertical plate with Newtonian heating has been investigated theoretically [3]. The stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below has been studied [4]. The nature of instability and some factors may have different effects, on viscoelastic fluids as compared to the Newtonian fluids. For example, the effect of a uniform rotation on the thermal instability of a Maxwell fluid has been studied and found that rotation has a destabilizing influence, in contrast to the stabilizing effect on a Newtonian fluid [5]. The thermal instability of a Maxwell fluid in hydromagnetics has been studied and have found that the magnetic field has stabilizing effect on viscoelastic fluid, just as in case of Newtonian fluid [6]. The thermal instability of a layer of Oldroydian fluid acted on by a uniform rotation has been analysed and found that rotation has destabilizing and stabilizing effects under certain conditions, in contrast to a Maxwell fluid where the effect is destabilizing [7,8]. In another study, the stability of a layer of an electrically conducting Oldroyd fluid in the presence of a magnetic field has been studied and found that the magnetic field has a stabilizing influence [9]. Many common materials such as paints, polymers, plastics and more exotic one such as silicic magma, saturated soils and the Earth's lithosphere behaves as viscoelastic fluids. Due to the growing use of these viscoelastic materials in various manufacturing and processing industries, in geophysical fluid dynamics, in chemical technology and in petroleum industry, considerable effort has been directed towards understanding their flow.

There are many elastico-viscous fluids that cannot be characterize by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of viscoelastic fluid is

Walters B' fluid [10]. It is reported that the mixture of polymethyl methacrylate and pyridine at 25^0C containing 30.5g of polymer per litre with density 0.98 g per litre behaves very nearly as the Walters B' viscoelastic fluid [11]. Polymers are used in the manufacture of spacecrafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastic engineering equipments, contact lens etc. Walters B' viscoelastic fluid forms the basis for the manufacture of many such important and useful products.

In many geophysical fluid dynamical problems encountered, the fluid is electrically conducting and a uniform magnetic field of the Earth pervades the system. A layer of such fluid heated from below under the action of magnetic field and rotation may find applications in geophysics, interior of the Earth, oceanography and the atmospheric physics. Keeping in mind the importance of viscoelastic fluids, convection in fluid layer heated from below, magnetic field and rotation; the present paper attempts to study the effect of uniform vertical magnetic field on Walters B' viscoelastic fluid heated from below in the presence of a uniform rotation.

2. Related Work

The flow of unsteady viscoelastic (Walters liquid B') conducting fluid through two porous concentric non-conducting infinite circular cylinders rotating with different angular velocities in the presence of a uniform axial magnetic field has been studied [12]. The stability of two superposed Walters B' viscoelastic liquids have been studied [13]. In another study, the Rayleigh-Taylor instability of two superposed conducting Walters B' elastico-viscous fluids in hydromagnetics has been given [14]. The effect of rotation on thermal instability in Walters elastico-viscous fluid has been studied and found that for stationary convection, rotation has a stabilizing effect [15]. The stability of plane interface separating the Walters B' viscoelastic superposed fluids of uniform densities in the presence of suspended particles has been considered [16].

3. Mathematical Formulation of the Problem

The present problem is studied using methods of linearized stability theory and normal mode analysis. First of all linearized perturbation equations relevant to the problem are obtained and then in section 3 the dispersion relation obtained by using normal analysis method.

Consider an infinite, horizontal, incompressible electrically conducting Walters B' viscoelastic fluid layer of thickenss d, heated from below so that, the temperatures and densities at the bottom surface z = 0 are T_0 and ρ_0 and at the upper surface $z = d$ are T_d and ρ_d respectively and that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The gravity $\vec{g}(0,0,-g)$, a uniform vertical magnetic field $\vec{H}(0,0,H)$ and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ act on the system.

Let $\vec{q}(u,v,w)$, p,ρ,T,υ and υ' denote the fluid velocity, pressure, density, temperature, kinematic viscosity and kinematic viscoelasticity, respectively and $\vec{r}=(x,y,z)$. Then the momentum balance, mass balance and energy balance equations of Walters B' viscoelastic fluid in the presence of magnetic field and rotation are

$$\frac{\partial\vec{q}}{\partial t}+(\vec{q}\cdot\nabla)\vec{q}=-\nabla\left(\frac{p}{\rho_0}-\frac{1}{2}\left|\vec{\Omega}\times\vec{r}\right|^2\right)+ \vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\nabla^2\vec{q} +\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}+2\left(\vec{q}\times\vec{\Omega}\right)\ , \tag{1}$$

$$\nabla\cdot\vec{q}=0\ , \tag{2}$$

$$\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T=\chi\nabla^2 T\ , \tag{3}$$

$$\nabla\cdot\vec{H}=0\ , \tag{4}$$

$$\frac{\partial\vec{H}}{\partial t}=\left(\vec{H}\cdot\nabla\right)\vec{q}+\eta\nabla^2\vec{H}\ . \tag{5}$$

The equation of state is

$$\rho=\rho_0[1-\alpha(T-T_0)], \tag{6}$$

where ρ_0, T_0 are respectively, the density and temperature of the fluid at the reference level z = 0 and α is the coefficient of thermal expansion. In writing equation (1), use has been made of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equations of motion except the external force term. The magnetic permeability μ_e, thermal diffusivity χ and electrical resistivity η are all assumed to be constants.

The steady solution is

$$\vec{q}=(0,0,0),\ T=T_0-\beta z\ ,\ \rho=\rho_0(1+\alpha\beta z). \tag{7}$$

Let $\vec{q}(u,v,w)$, δp, $\delta\rho$, θ and $\vec{h}(h_x,h_y,h_z)$ denote respectively the perturbations in velocity $\vec{q}$ (initially zero), pressure p, density ρ, temperature T and the magnetic field $\vec{H}(0,0,H)$. The change in density $\delta\rho$, caused by the perturbation θ in temperature, is given by

$$\rho+\delta\rho=\rho_0[1-\alpha(T+\theta-T_0)]=\rho-\alpha\rho_0\theta$$

i.e.

$$\delta\rho=-\alpha\rho_0\theta\ . \tag{8}$$

Then the linearized perturbation equations for Walters B' viscoelastic fluid are

$$\frac{\partial\vec{q}}{\partial t}=-\frac{1}{\rho_0}(\nabla\delta p)-\vec{g}\alpha\theta+\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\nabla^2\vec{q} +\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}+2\left(\vec{q}\times\vec{\Omega}\right) \tag{9}$$

$$\nabla\cdot\vec{q}=0\ , \tag{10}$$

$$\frac{\partial\theta}{\partial t}=\beta w+\chi\nabla^2\theta\ , \tag{11}$$

$$\nabla\cdot\vec{h}=0\ , \tag{12}$$

$$\frac{\partial \vec{h}}{\partial t} = \left(\vec{H} \cdot \nabla\right)\vec{q} + \eta \nabla^2 \vec{h} \,. \quad (13)$$

Within the framework of Boussinesq approximation, equations (9) - (13), become

$$\frac{\partial}{\partial t}\nabla^2 w = \left(\upsilon - \upsilon' \frac{\partial}{\partial t}\right)\nabla^4 w + \frac{\mu_e}{4\pi\rho_0}\nabla^2 \left(\frac{\nabla h_z}{\partial z}\right) + g\alpha\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) - 2\Omega \frac{\partial \zeta}{\partial z} \quad (14)$$

$$\frac{\partial \zeta}{\partial t} = \left(\upsilon - \upsilon' \frac{\partial}{\partial t}\right)\nabla^2 \zeta + 2\Omega \frac{\partial w}{\partial z} - \frac{\mu_e H}{4\pi\rho_0}\frac{\partial \xi}{\partial z} \,, \quad (15)$$

$$\left[\frac{\partial}{\partial t} - \chi \nabla^2\right]\theta = \beta w \,, \quad (16)$$

$$\left[\frac{\partial}{\partial t} - \eta \nabla^2\right]h_z = H\frac{\partial w}{\partial z} \,, \quad (17)$$

$$\left[\frac{\partial}{\partial t} - \eta \nabla^2\right]\xi = H\frac{\partial \zeta}{\partial z} \,, \quad (18)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}; \xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$, stand for the z-components of vorticity and current density, respectively.

4. The Dispersion Relation

We decompose the disturbances into normal modes and assume that the perturbed quantities are of the form

$$[w, \theta, h_z, \zeta, \xi] = [W(z), \Theta(z), K(z), Z(z), X(z)] \exp\left(ik_x x + ik_y y + nt\right) \quad (19)$$

where k_x, k_y are the wave numbers along x-and y-directions, respectively, $k = \left(k_x^2 + k_y^2\right)^{1/2}$ is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using expression (19), equations (14) - (18) in non-dimensional form transform to

$$\left[\sigma\left(D^2 - a^2\right)W + \left(\frac{g\alpha d^2}{\upsilon}\right)a^2\Theta + \frac{2\Omega d^3}{\upsilon}DZ - \frac{\mu_e H d}{4\pi\rho_0 \upsilon}\left(D^2 - a^2\right)DK\right] = [1 - F\sigma]\left(D^2 - a^2\right)^2 W \quad (20)$$

$$\left[\{1 - F\sigma\}\left(D^2 - a^2\right) - \sigma\right]Z = -\frac{2\Omega d}{\upsilon}DW - \left(\frac{\mu_e H d}{4\pi\rho_0 \upsilon}\right)DX \,, \quad (21)$$

$$\left[D^2 - a^2 - p_1\sigma\right]\Theta = -\left(\frac{\beta d^2}{\chi}\right)W \,, \quad (22)$$

$$\left[D^2 - a^2 - p_2\sigma\right]K = -\left(\frac{Hd}{\eta}\right)DW \,, \quad (23)$$

$$\left[D^2 - a^2 - p_2\sigma\right]X = -\left(\frac{Hd}{\eta}\right)DZ \,, \quad (24)$$

where we have introduced new co-ordinates $(x', y', z') = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right)$ in new units of length d and $D = \frac{d}{dz'}$. For convenience, the dashes are dropped hereafter. Also we have put $a = kd$, $\sigma = \frac{nd^2}{\upsilon}, F = \frac{\upsilon'}{d^2}$, $p_1 = \frac{\upsilon}{\chi}$ is the Prandtl number and $p_2 = \frac{\upsilon}{\eta}$ is the magnetic Prandtl number.

Here we consider the case where both the boundaries are free as well as perfect conductors of heat, while the adjoining medium is also perfectly conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres and in certain geophysical situations where it is most appropriate [17]. However, the case of two free boundaries allows us to obtain analytical solution without affecting the essential features of the problem. The appropriate boundary conditions, with respect to which equations (20) - (24) must be solved, are

$W = D^2W = 0, DZ = 0, \Theta = 0$ *at* $z = 0$ and $z = 1$, $DX = 0$, $K = 0$ on a perfectly conducting boundary. (25)

Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for z = 0 and z = 1, and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z \,, \quad (26)$$

where W_0 is a constant.

Eliminating Θ, K, Z and X between equations (20) - (24) and substituting relation (26) in the resultant equation, we obtain the dispersion relation

$$R_1 = \left(\frac{1+x}{x}\right)\frac{\left[\frac{\left\{\left(1 - iF_1\sigma_1\pi^2\right)(1+x) + i\sigma_1\right\}}{\{1 + x + i\sigma_1 p_2\} + Q_1}\right][1 + x + i\sigma_1 p_1]}{(1 + x + i\sigma_1 p_2)} + \frac{\frac{T_1(1 + x + i\sigma_1 p_2)}{(1 + x + i\sigma_1 p_1)}}{\left\{x\left[\frac{\left\{\left(1 - iF_1\sigma_1\pi^2\right)(1+x) + i\sigma_1\right\}}{\{1 + x + i\sigma_1 p_2\} + Q_1}\right]\right\}} \quad (27)$$

where $R = \frac{g\alpha\beta d^4}{\upsilon\chi}, Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0 \upsilon\eta}, T_A = \frac{4\Omega^2 d^4}{\upsilon^2}$, stand for the Rayleigh-number, the Chandrasekhar number, the Taylor number, respectively and we have also put $x = \frac{a^2}{\pi^2}$, $R_1 = \frac{R}{\pi^4}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $F_1 = \pi^2 F$, $T_1 = \frac{T_A}{\pi^4}$, $Q_1 = \frac{Q}{\pi^2}$ and $i = \sqrt{-1}$.

5. Important Theorems and Discussion

Theorem 1: For stationary convection case:
(I) The viscoelasticity parameter F vanishes with σ and Walters B' viscoelastic fluid behaves like an ordinary Newtonian fluid.
(II) Rotation postpones the onset of convection i.e. rotation has a stabilizing effect on the system.
(III The magnetic field has both stabilizing and destabilizing effects on the system.
Proof: When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (27) reduces to

$$R_1 = \left(\frac{1+x}{x}\right)\left[(1+x)^2 + Q_1\right] + \frac{T_1(1+x)^2}{x\left[(1+x)^2 + Q_1\right]}, \tag{28}$$

To investigate the effects of rotation and magnetic field, we examine the behavior of $\frac{dR_1}{dT_1}$ and $\frac{dR_1}{dQ_1}$ analytically.
(I) It is evident from relation (28) that for the stationary convection, the viscoelasticity parameter F vanishes with σ and Walters B' viscoelastic fluid behaves like an ordinary Newtonian fluid.
(I) (II) Equation (28) yields

$$\frac{dR_1}{dT_1} = \frac{(1+x)^2}{x\left[(1+x)^2 + Q_1\right]}, \tag{29}$$

which is always positive. The rotation, therefore, has a stabilizing effect on the system for the case of stationary convection.
(III) It is also evident from equation (28) that

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{x} - \frac{T_1(1+x)^2}{x\left[(1+x)^2 + Q_1\right]^2} \tag{30}$$

It is clear from equation (30) that, for stationary convection, $\frac{dR_1}{dQ_1}$ may be positive as well as negative, thus, the magnetic field has both stabilizing and destabilizing effects on the system.
Theorem 2: The system is stable or unstable.
Proof: Multiplying equation (20) by W^*, the complex conjugate of W, integrating the resulting equation over the range of z and using equations (21) - (24) together with the boundary conditions (25), we obtain

$$\begin{aligned} &-\sigma I_1 + \frac{g\alpha\chi a^2}{\upsilon\beta}\left(I_2 + p_1\sigma^* I_3\right) - \frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}\left(I_6 + p_2\sigma I_7\right) - \frac{\mu_e\eta}{4\pi\rho_0\upsilon} \\ &- d^2\left(1 - F\sigma^*\right)I_4 - d^2\sigma^* I_5 - \left(I_8 + p_2\sigma^* I_9\right) = (1 - F\sigma)I_{10} \end{aligned}, \tag{31}$$

where

$$I_1 = \int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz,$$

$$I_2 = \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz,$$

$$I_3 = \int_0^1 \left(|\Theta|^2\right)dz, \qquad I_4 = \int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz,$$

$$I_5 = \int_0^1 \left(|Z|^2\right)dz, \qquad I_6 = \int_0^1 \left(|DX|^2 + a^2|X|^2\right)dz,$$

$$I_7 = \int_0^1 \left(|X|^2\right)dz,$$

$$I_8 = \int_0^1 \left(\left|D^2K\right|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz,$$

$$I_9 = \int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz,$$

$$I_{10} = \int_0^1 \left(\left|D^2W\right|^2 + 2a^2|DW|^2 + a^4|W|^2\right)dz, \tag{32}$$

and σ* is the complex conjugate of σ. The integrals $I_1, \ldots, I_{10}$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$, where σ_r, σ_i are real and equating the real and imaginary parts of equation (31), we obtain

$$\begin{aligned} &\sigma_r\left[\begin{array}{l} -I_1 + \frac{g\alpha\chi a^2}{\upsilon\beta}p_1I_3 + d^2FI_4 - d^2I_5 \\ -\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}p_2I_7 - \frac{\mu_e\eta}{4\pi\rho_0\upsilon}p_2I_9 + FI_{10} \end{array}\right] \\ &= -\frac{g\alpha\chi a^2}{\upsilon\beta}I_2 + d^2I_4 + \frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}I_6 \\ &+ \frac{\mu_e\eta}{4\pi\rho_0\upsilon}I_8 + I_{10} \end{aligned}, \tag{33}$$

$$\sigma_i\left[\begin{array}{l} I_1 + \frac{g\alpha\chi a^2}{\upsilon\beta}p_1I_3 + d^2FI_4 - d^2I_5 \\ +\frac{\mu_e d^2\eta}{4\pi\rho_0\upsilon}p_2I_7 - \frac{\mu_e\eta}{4\pi\rho_0\upsilon}p_2I_9 - FI_{10} \end{array}\right] = 0. \tag{34}$$

It is evident from equation (33) that σ_r is either positive or negative. The system is, therefore, stable or unstable.
Theorem 3: The modes may be either oscillatory or non-oscillatory in contrast to the non-magneto-rotatory case.
Proof: Equation (34) yields that σ_i may be either zero or non-zero, which means that the modes may be either non-oscillatory or oscillatory. In the absence of rotation and magnetic field, equation (34) reduces to

$$\sigma_i\left[I_1 + \frac{g\alpha\chi a^2}{\upsilon\beta}p_1I_3\right] = 0,$$

and the terms in brackets are positive definite. Thus, $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for Walters B' viscoelastic fluid heated from below. Thus, rotation, magnetic field and viscoelasticity introduce oscillatory modes (as σ_i may not be zero) in the system which was non-existent in their absence.

Theorem 4: The system is stable for

$$\frac{\mathbf{g\alpha\kappa}}{\mathbf{\nu\beta}} \leq (\mathbf{1 - 2\sigma_r F}) \frac{\mathbf{27\pi^4}}{\mathbf{4}}$$

and under the conditions

$$\frac{\mathbf{g\alpha\kappa}}{\mathbf{\nu\beta}} > (\mathbf{1 - 2\sigma_r F}) \frac{\mathbf{27\pi^4}}{\mathbf{4}} \textbf{ and } \mathbf{0 \leq \sigma_r \leq \frac{1}{2F}}$$

the system becomes unstable.

Proof: From equation (34), it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If $\sigma_i \neq 0$, then equation (34) gives

$$\frac{g\alpha\chi a^2}{\upsilon\beta} p_1 I_3 + d^2 F I_4 - d^2 I_5 - \frac{\mu_e \eta}{4\pi\rho_0\upsilon} p_2 I_9 = -I_1 - \frac{\mu_e d^2 \eta}{4\pi\rho_0\upsilon} p_2 I_7 + F I_{10}. \tag{35}$$

Substituting in equation (33), we have

$$I_{10} + \frac{\mu_e \eta}{4\pi\rho_0\upsilon} I_8 + \frac{\mu_e d^2 \eta}{4\pi\rho_0\upsilon} I_6 + d^2 I_4 + 2\sigma_r \left[I_1 + \frac{\mu_e d^2 \eta}{4\pi\rho_0\upsilon} p_2 I_7 - F I_{10} \right] = \frac{g\alpha\chi a^2}{\upsilon\beta} I_2, \tag{36}$$

Equation (36) on using Rayleigh-Ritz inequality, gives

$$[1 - 2\sigma_r F] \frac{(\pi^2 + a^2)^3}{a^2} \int_0^1 |W|^2 dz + \frac{(\pi^2 + a^2)}{a^2} \left\{ \begin{array}{l} \frac{\mu_e \eta}{4\pi\rho_0\upsilon} I_8 + \frac{\mu_e d^2 \eta}{4\pi\rho_0\upsilon} I_6 + d^2 I_4 + \\ 2\sigma_r \left[I_1 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 \nu} p_2 I_7 \right] \end{array} \right\} \leq \frac{g\alpha\chi}{\nu\beta} \int_0^1 |W|^2 \, dz. \tag{37}$$

Therefore, it follows from equation (37) that

$$\left[(1 - 2\sigma_r F) \frac{27\pi^4}{4} - \frac{g\alpha\chi}{\nu\beta} \right] \int_0^1 |W|^2 \, dz + \frac{(\pi^2 + a^2)}{a^2} \left\{ \begin{array}{l} \frac{\mu_e \eta}{4\pi\rho_0\upsilon} I_8 + \frac{\mu_e d^2 \eta}{4\pi\rho_0\upsilon} I_6 + d^2 I_4 + \\ 2\sigma_r \left[I_1 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 \nu} p_2 I_7 + F I_{10} \right] \end{array} \right\} \leq 0, \tag{38}$$

since minimum value of $\frac{(\pi^2 + a^2)^3}{a^2}$ w.r.t. a^2 is $\frac{27\pi^4}{4}$.

Now, let $\sigma_r \geq 0$, we necessarily have from equation (38), that

$$\frac{g\alpha\chi}{\nu\beta} > (1 - 2\sigma_r F) \frac{27\pi^4}{4} \quad \textit{and} \ 0 \leq \sigma_r \leq \frac{1}{2F}, \tag{39}$$

Hence, if $\frac{g\alpha\chi}{\nu\beta} \leq (1 - 2\sigma_r F) \frac{27\pi^4}{4}$ (40)

then $\sigma_r < 0$. Therefore, the system is stable.

Therefore, under condition (40), the system is stable and under the conditions (39), the system becomes unstable.

6. Conclusions and Future Scope

A layer of Newtonian fluid heated from below under varying assumptions of hydrodynamics in the presence and absence of rotation and magnetic field has been studied [1]. With the growing importance of viscoelastic fluids in chemical engineering, modern technology and industry, the investigations on such fluids are desirable. The Walters B' fluid is one such important viscoelastic fluid. Keeping in mind the importance of viscoelastic fluids, the present paper considering the effect of rotation on the Walters B' viscoelastic fluid heated from below in the presence of a uniform vertical magnetic field.

The main conclusions from the analysis of this paper are as follows:

1. For the case of stationary convection the following observations are made:
   a. The viscoelasticity parameter F vanishes with σ and Walters B' viscoelastic fluid behaves like an ordinary Newtonian fluid.
   b. The rotation postpones the onset of convection i.e. rotation has a stabilizing effect on the system.
   c. The magnetic field has both stabilizing and destabilizing effects on the system.
2. It is found that rotation and magnetic field introduce oscillatory modes in the system which was non-existent in their absence.

1. It is observed that the system is stable for $\frac{g\alpha\kappa}{\nu\beta} \leq (1 - 2\sigma_r F) \frac{27\pi^4}{4}$ and under the conditions $\frac{g\alpha\kappa}{\nu\beta} > (1 - 2\sigma_r F) \frac{27\pi^4}{4}$ and $0 \leq \sigma_r \leq \frac{1}{2F}$, the system becomes unstable.
2. However, further the study of stability, bifurcation, and pattern formation in thermal convection for viscoelastic fluids is an exciting area of research. Understanding these complex behaviors can have significant implications for natural convection, pattern formation in industrial flows, and optimizing heat exchange systems. Also the combination of Walter's viscoelastic fluid with nanoparticles (nanofluids) could enhance heat transfer characteristics. Thermal convection analysis in these hybrid fluids can lead to the design of high-performance cooling systems for electronics and advanced engineering applications.

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References

[1] S.Chandrasekhar, "Hydrodynamic and Hydromagnetic Stability," New York: Dover Publication, **1981**.

[2] H.A. Abdul-Bari, R.T.A. Al-Rubai, "Influence of Number in Turbulent and Laminar Region in Parallel-plate Vertical Channels," *Asian J. Applied Sci*., Vol.**1**, pp.**265-285**, **2008**.

[3] M. Narahari, A. Ishak, "Radiation Effects on Free Convection Flow Near a Moving Vertical Plate with Newtonian Heating," *J. Applied Sci*., Vol.**11**, pp.**1096-1104**, **2011**.

[4] C.M. Vest, V.S. Arpaci., "Overstability of a Viscoelastic Fluid Layer Heated From Below," *J. Fluid Mech*., Vol. **36**, pp. **613-619**, **1969**.

[5] P.K. Bhatia, J.M. Steiner, "Convective Instability in a Rotating Viscoelastic Fluid Layer," *Z. Angew. Math. Mech*., Vol.**52,** pp.**321-324**, **1972**.

[6] P.K. Bhatia, J.M. Steiner, "Thermal Instability in a Viscoelastic Fluid Layer in Hydromagnetics," *J. Math. Anal. Appl*., Vol.**41,** pp**.271-283**, **1973**.

[7] J.G. Oldroyd, "Non-Newtonian Effects in Steady Motion of Some Idealized Elastico-Viscous Liquids," *Proc. Roy. Soc. London*, Vol.**A245,** pp.**278-297**, **1958**.

[8] R.C. Sharma, "Effect of Rotation on Thermal Instability of a Viscoelastic Fluid," *Acta Phys. Hung*., Vol.**40,** pp.**11-17**, **1976**.

[9] R.C. Sharma, "Thermal Instability in a Viscoelastic Fluid in Hydromagnetics," *Acta Phys. Hung*., Vol. **38,** pp. **293-298**, **1975**.

[10] K. Waters, "The Motion of Elastico-Viscous Liquid Contained Between Coaxial Cylinders," *J. Mech. Appl. Math*., Vol.**13**, pp.**444-453**, **1960**.

[11] K. Walters, "The Solution of Flow Problems in Case of Materials with Memory," *J. Mecanique*, Vol.**1**, pp.**469-479**, **1962**.

[12] G. Chakraborty, P.R. Sengupta, "MHD Flow of Unsteady Viscoelastic (Walters liquid B') Conducting Fluid Between Two Concentric Circular Cylinders," *Proc. Nat. Acad. Sciences India*, Vol.**64**, pp.**75-80**, **1994**.

[13] R.C. Sharma, P. Kumar, "On the Stability of Two Superposed Walters Elastico-Viscous Liquid B'," *Czech. J. Phys*., Vol.**47**, pp.**197-204**, **1997**.

[14] R.C. Sharma, P. Kumar, "Rayleigh-Taylor Instability of Two Superposed Walters B' Elastico-Viscous Fluids in Hydromagnetics," *Proc. Nat. Acad. Sciences India*, Vol.**68**, pp.**151-161**, **1998**.

[15] P. Kumar, "Effect of Rotation on Thermal Instability in Walters B' Elastico-Viscous Fluid," *Proc. Nat. Acad. Sciences India*, Vol.**71**, pp.**33-41**, **2001**.

[16] P. Kumar, H. Mohan, G.J. Singh, "Stability of Two Superposed Viscoelastic Fluid-Particle Mixtures," *Z. Angew. Math. Mech*., Vol.**86**, pp.**72-77**, **2006**.

[17] E.A. Spiegel, "Convective Instability in a Compressible Atmosphere, " *Astrophys. J.*, Vol.**141,** pp.**1068-1090**, **1965**.

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