**International Journal of Scientific Research in Physics and Applied Sciences** **Research Paper**
Vol.7, Issue.4, pp.33-38, August (2019) **E-ISSN:** 2348-3423
*DOI: https://doi.org/10.26438/ijsrpas/v7i4.3338*

# Reduction of Quantum Phase Fluctuations in Multi-Wave Mixing Via Measured Phase Operators

**Savita Gill**

Department of Applied Science, University Institute of Engineering and Technology, Kurukshetra 136 119, Haryana, India

*Corresponding Author: Savita2015@kuk.ac.in*

**Available online at: www.isroset.org**

Received: 05/Jul/2019, Accepted: 22/Jul/2019, Online: 31/Aug/2019

***Abstract*-**Quantum phase parameters have been studied in Multi-wave mixing using Barnett-Pegg formalism. It is shown that reduction of Carruther and Nieto symmetric quantum phase fluctuation parameter U with respect to its coherent state value corresponds to an antibunched state and thus phase fluctuation parameter can be used as a measure of nonclassicality of a state. Further we have compared the measured values of Carruther and Nieto quantum phase parameters U, Q and S and found the reduction of U in five waves mixing optical process directly relates to antibunching.

***Keywords**:* Quantum phase, Nonclassicality, Multiwave mixing.

## I. INTRODUCTION

Squeezing and antibunching have no classical counterpart and are called non-classical states. Antibunching or Sub-poissonian light has a narrower photon number distribution than for Poissonian statistics. The 'Hermitian quantum phase operators' have some ambivalence [1-3] which leads to lot of unalike formalisms [4-6] of phase problem. Out of these formalism Susskind-Glogower (SG) [4] and Barnett-Pegg (BP) [5] formalism played major part in phase fluctuation. Use of the measured phase operators of BP formalism provides easier calculations than SG formalism. Phase operators in various non linear processes have been studied using SG formalism [7-11] as well as BP formalism [12-16]. Carruther and Nieto [11] introduced symmetric quantum phase parameters to study phase fluctuation. With the observation of quantum phase fluctuation in quantum computation [17,18], super-conductivity [19] and photon added coherent state [20], there has been an increase in the study of quantum phase parameters.

In the present work Barnett-Pegg (BP) [5] formalism has been applied to study quantum phase operators in multiwave mixing using Carruther and Nieto phase parameters. In section II and III we briefly explain phase fluctuation parameters U, S and Q and presented short time approximated operator solution upto second order of five wave mixing optical process with physical meaning of U parameter. Finally section IV is dedicated to conclusion.

## II. MEASUREMENT OF QUANTUM PHASE FLUCTUATION PARAMETERS

Barnett and Pegg [5] defined the exponential of phase operator E and its Hermitian conjugate $E^{\dagger}$ as

$$E = \left(\overline{N} + \frac{1}{2}\right)^{-\frac{1}{2}} a(t)$$

$$E^{\dagger} = \left(\overline{N} + \frac{1}{2}\right)^{-\frac{1}{2}} a^{\dagger}(t) \qquad (1)$$

Where $\overline{N}$ is the mean photon number in the coherent state.

 

Sine and cosine operators are defined as [5]

$$C=\frac{1}{2}(E+E^{\dagger})$$

$$S=-\frac{i}{2}(E-E^{\dagger}) \tag{2}$$

And satisfy the following relations

$$\langle C\rangle^{2}+\langle S\rangle^{2}=1 \tag{3}$$

$$[C,S]=\frac{i}{2}\left(\overline{N}+\frac{1}{2}\right)^{-\frac{1}{2}} \tag{4}$$

So

$$(\Delta C)^{2}(\Delta S)^{2}\geq\frac{1}{16}\frac{1}{(\overline{N}+\frac{1}{2})} \tag{5}$$

To measure quantum phase fluctuation, Carruthers and Nieto [11] had introduced U, S and Q parameters in the following way:

$$U\left(\theta,t,|\alpha|^{2}\right)=(\Delta N)^{2}[(\Delta S)^{2}+(\Delta C)^{2}]/[\langle S\rangle^{2}+\langle C\rangle^{2}] \tag{6}$$

$$S(\theta,t,|\alpha|^{2})=(\Delta N)^{2}(\Delta S)^{2} \tag{7}$$

And

$$Q(\theta,t,|\alpha|^{2})=S(\theta,t,|\alpha|^{2})/\langle C\rangle^{2} \tag{8}$$

Where $\theta$ is the phase of input coherent state $|\alpha\rangle$ , t is the interaction time and $|\alpha|^{2}$ is average photon number in coherent state.

## III. QUANTUM PHASE IN FIVE WAVE INTERACTION PROCESS

The Hamiltonian for five wave interaction process involving two pump photons of frequency $\omega_1$ and emitted photons of frequency $\omega_2$ and $\omega_3$ is given as

$$H=\omega_1 a^{+}a+\omega_2 b^{+}b+\omega_3 c^{+}c+g\left(a^{2}b^{+2}c^{+}+a^{+2}b^{2}c\right) \tag{9}$$

Where g is a coupling constant, $a^{\dagger}(a)$, $b^{\dagger}(b)$ and $c^{\dagger}(c)$ are creation (annihilation) operators, respectively. $A=a\exp i\omega_1 t$, $B=b\exp i\omega_2 t$, $C=c\exp i\omega_3 t$ are operators at frequencies $\omega_1$, $\omega_2$ and $\omega_3$ respectively.

To study quantum phase fluctuation a coherent state $|\alpha\rangle$ is used as pump for mode A and before the interaction process there was no photon in signal mode B and stokes mode C i.e

$$|\psi\rangle=|\alpha\rangle_A|0\rangle_B|0\rangle_C$$

The Heisenberg equation of motion for fundamental mode $A$ is given as $(\hbar=1)$

$$\frac{dA}{dt}=\frac{\partial A}{\partial t}+i[H,A] \tag{10}$$

Now using Taylor's series expansion and short time approximation, $A(t)$ can be expressed as

$$A(t) = A - 2igtA^\dagger B^2 C + g^2t^2 \left(2AN_B{}^2 N_C - A^\dagger A^2 N_B{}^2 - 4A^\dagger A^2 N_B N_C \right.$$
$$\left. -4A^\dagger A^2 N_B - 2A^\dagger A^2 N_C - 2A^\dagger A^2 \right) \tag{11}$$

Where $N_B = B^\dagger B$ and $N_C = C^\dagger C$ .

From equation (11) average number of photons can be expressed as

$$N(t) = A^\dagger A - 2igt(A^{\dagger 2} B^2 C - A^2 B^{\dagger 2} C^\dagger) + 4g^2t^2(2A^\dagger AN_B{}^2 N_C + N_B{}^2 N_C)$$
$$-2g^2t^2(A^{\dagger 2} A^2 N_B{}^2 + 4A^{\dagger 2} A^2 N_B N_C + 4A^{\dagger 2} A^2 N_B + 2A^{\dagger 2} A^2 N_C + 2A^{\dagger 2} A^2) \tag{12}$$

The expectation value of $N(t)$ is

$$\langle N \rangle = |\alpha|^2 - 4g^2t^2 |\alpha|^4 \tag{13}$$

$$\langle N \rangle^2 = |\alpha|^4 - 8g^2t^2 |\alpha|^6 \tag{14}$$

$$\langle N^2 \rangle = |\alpha|^4 + |\alpha|^2 - g^2t^2(8|\alpha|^6 + 16|\alpha|^4) \tag{15}$$

Using equation (14) and (15), we obtain

$$(\Delta N)^2 = \langle N^2 \rangle - \langle N \rangle^2$$

$$(\Delta N)^2 = |\alpha|^2 - g^2t^2 \left(16|\alpha|^4\right) \tag{16}$$

**Condition of sub-Poissonian photon statistics is given as**

$$d = (\Delta N)^2 - \langle N \rangle < 0 \tag{17}$$

Using (13) and (16), we get

$$d = -12g^2t^2 |\alpha|^4 \tag{18}$$

We are getting negative value which shows that that photon statistics is sub-Poissonian or antibunched light.
By substituting (11) in (2), we obtain

$$C = \frac{1}{2}(N + \frac{1}{2})^{-\frac{1}{2}}[A + A^\dagger - 2igtA^\dagger B^2 C + 2igtAB^{\dagger 2} C^\dagger + g^2t^2(2AN_B{}^2 N_C - A^\dagger A^2 N_B{}^2$$
$$-4A^\dagger A^2 N_B N_C - 4A^\dagger A^2 N_B - 2A^\dagger A^2 N_C + 2A^\dagger N_B{}^2 N_C - A^{\dagger 2} AN_B{}^2 - 4A^{\dagger 2} AN_B N_C$$
$$-4A^{\dagger 2} AN_B - 2A^{\dagger 2} AN_C - 2A^\dagger A^2 - 2A^{\dagger 2} A)] \tag{19}$$

And

$$S = -\frac{i}{2}(N + \frac{1}{2})^{-\frac{1}{2}}[A - A^\dagger - 2igtA^\dagger B^2 C - 2igtAB^{\dagger 2} C^\dagger + g^2t^2(2AN_B{}^2 N_C$$
$$-A^\dagger A^2 N_B{}^2 - 4A^\dagger A^2 N_B N_C - 4A^\dagger A^2 N_B - 2A^\dagger A^2 N_C - 2A^\dagger N_B{}^2 N_C + A^{\dagger 2} AN_B{}^2$$
$$+4A^{\dagger 2} AN_B N_C + 4A^{\dagger 2} AN_B + 2A^{\dagger 2} AN_C - 2A^\dagger A^2 + 2A^{\dagger 2} A)] \tag{20}$$

The expectation value of C and S operators of equation (19) and (20) are

$$\langle C \rangle = \frac{1}{2}[(N + \frac{1}{2})^{-\frac{1}{2}}\{\alpha + \alpha^* - g^2t^2(2|\alpha|^2 \alpha + 2|\alpha|^2 \alpha^*)\}] \tag{21}$$

$$\langle S \rangle = -\frac{i}{2}[(\overline{N} + \frac{1}{2})^{-\frac{1}{2}}\{\alpha - \alpha^* - g^2t^2(2|\alpha|^2 \alpha - 2|\alpha|^2 \alpha^*)\}] \tag{22}$$

 

Then square of expectation value of C and S are

$$\langle C\rangle^2 = \frac{1}{4}(N+\frac{1}{2})^{-1}[\alpha^2+\alpha^{*2}+2|\alpha|^2 - g^2t^2(4|\alpha|^2\alpha^2+4|\alpha|^2\alpha^{*2}+8|\alpha|^4) \quad (23)$$

$$\langle S\rangle^2 = -\frac{1}{4}(N+\frac{1}{2})^{-1}[\alpha^2+\alpha^{*2}-2|\alpha|^2 - g^2t^2(4|\alpha|^2\alpha^2+4|\alpha|^2\alpha^{*2}-8|\alpha|^4) \quad (24)$$

Similarly

$$\langle C^2\rangle = \frac{1}{4}(N+\frac{1}{2})^{-1}[\alpha^2+\alpha^{*2}+2|\alpha|^2+1-g^2t^2(2\alpha^2+2\alpha^{*2}+4|\alpha|^2\alpha^2 +4|\alpha|^2\alpha^{*2}+8|\alpha|^4+8|\alpha|^2) \quad (25)$$

$$\langle S^2\rangle = -\frac{1}{4}(N+\frac{1}{2})^{-1}[\alpha^2+\alpha^{*2}-2|\alpha|^2-1-g^2t^2(2\alpha^2+2\alpha^{*2}+4|\alpha|^2\alpha^2 +4|\alpha|^2\alpha^{*2}-8|\alpha|^4-8|\alpha|^2)] \quad (26)$$

Using equations (23)-(26), second order variances is expressed as

$$(\Delta C)^2 = \frac{1}{4}(N+\frac{1}{2})^{-1}[1-g^2t^2(2\alpha^2+2\alpha^{*2}+8|\alpha|^2)] \quad (27)$$

And

$$(\Delta S)^2 = -\frac{1}{4}(\overline{N}+\frac{1}{2})^{-1}[-1-g^2t^2(2\alpha^2+2\alpha^{*2}-8|\alpha|^2)] \quad (28)$$

Now equations (6)-(8) can be expressed as

$$U(\theta,t,|\alpha|^2) = \frac{1}{2}\left\{\frac{1-24g^2t^2|\alpha|^2}{1-4g^2t^2|\alpha|^2}\right\} \quad (29)$$

$$S(\theta,t,|\alpha|^2) = \frac{1}{4}(|\alpha|^2-4g^2t^2|\alpha|^4+\frac{1}{2})^{-1}[|\alpha|^2+4|\alpha|^4 g^2t^2(\cos 2\theta-6)] \quad (30)$$

And

$$Q(\theta,t,|\alpha|^2) = \frac{1+4|\alpha|^2 g^2t^2(\cos 2\theta-6)}{2(\cos 2\theta+1)(1-4|\alpha|^2 g^2t^2)} \quad (31)$$

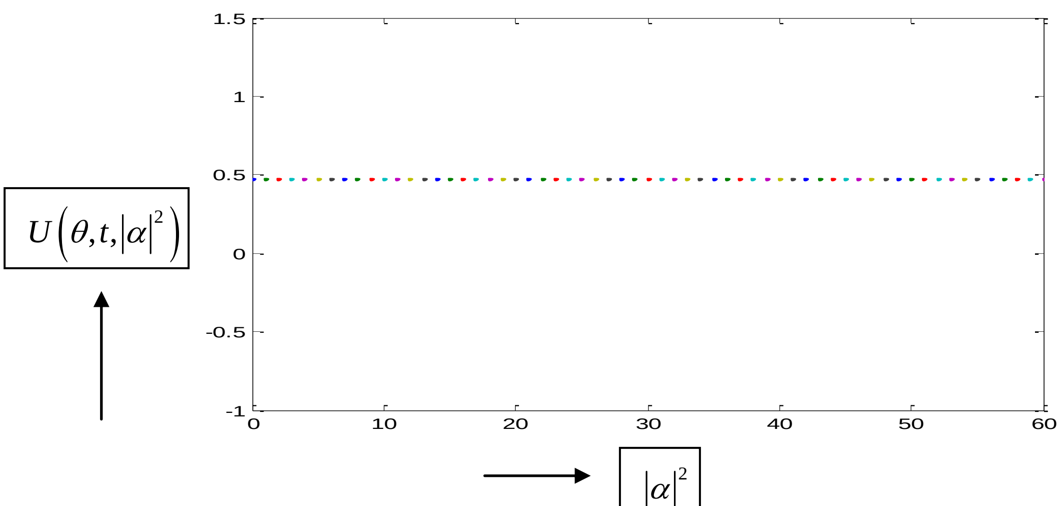


Figure 1. Variation of parameter U with N in five wave mixing process (taking $g^2t^2 = 10^{-4}$ and $\theta = 0$ for maximum squeezing)

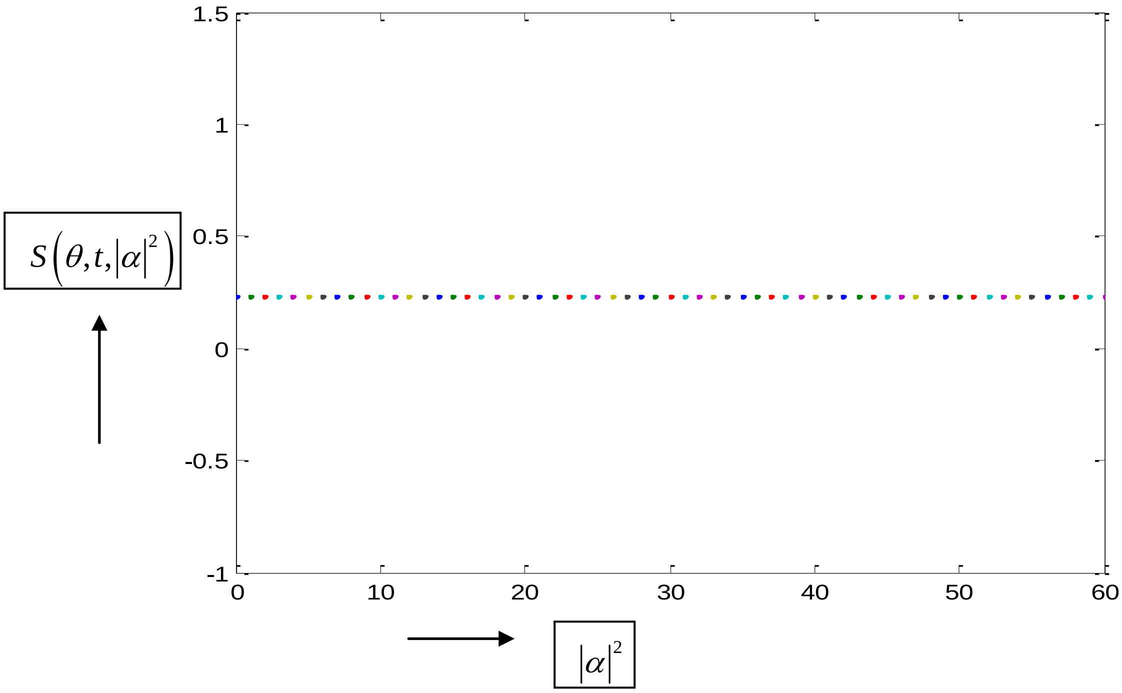


Figure 2. Variation of parameter S with N in five wave mixing process (taking $g^2t^2 = 10^{-4}$ and $\theta = 0$ for maximum squeezing)

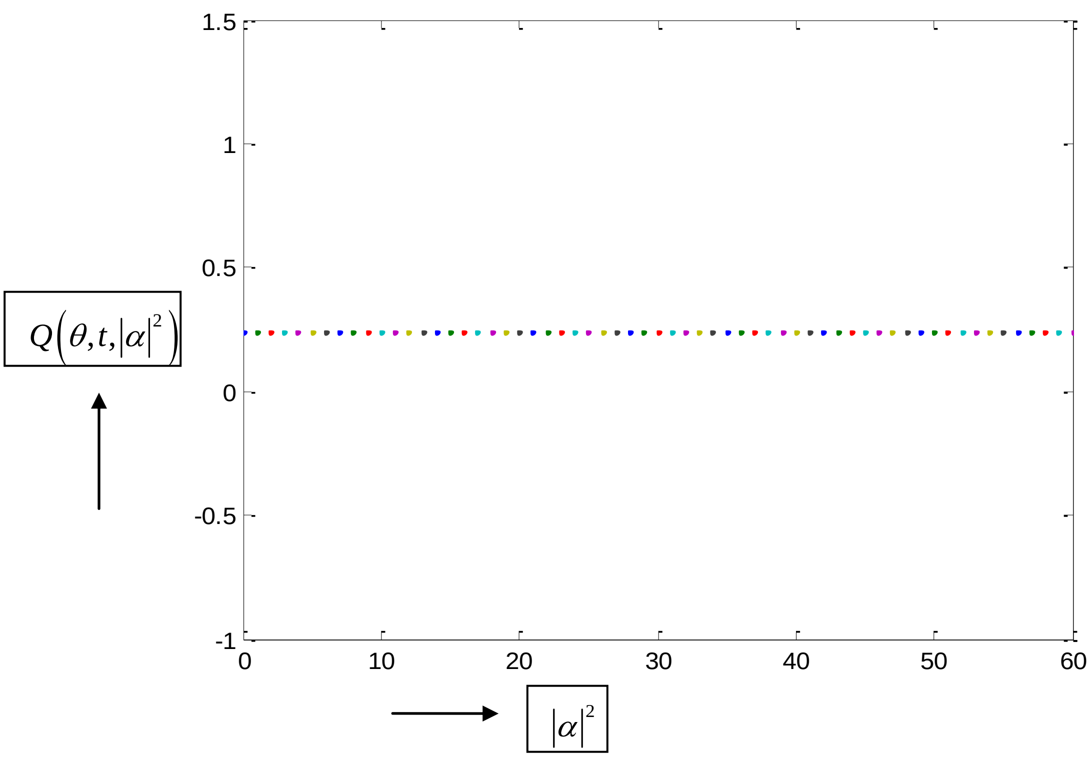


Figure 3. Variation of parameter Q with N in five wave mixing process (taking $g^2t^2 = 10^{-4}$ and $\theta = 0$ for maximum squeezing)

 

Thus from (18) and (29) it is clear that decrease in the value of phase parameter U with increase in number of photons in coherent state with respect to its Poissonian state value leads to antibunching phenomenon [21]. Now $U_O = 1/2$, $S_O = 1/4|\alpha|^2 (|\alpha|^2 + 1/2)^{-1}$ and $Q_O = 1/2\cos 2\theta$ are the initial values of measured phase operators and gives information about phase of the input coherent light.

## IV. CONCLUSION AND FUTURE SCOPE

The results obtained for symmetric uncertainty product U from (29) clearly shows the decrease in value of parameter U with respect to its classical (Poissonian) state value as we increase the initial photon number $|\alpha|^2$ It is also shown that the parameter U is independent of $\theta$ while S and Q parameters can be tuned by varying the values of time of interaction t and phase of the input coherent state $\theta$. Hence the present work directly relates the reduced value of measured phase parameter U with antibunching (Sub-Poissonian). Thus we can conclude that reduction of phase fluctuation parameters increases with increasing non-classicality of the system and it can be used as a criterion for single photon source that is the primary requirement for quantum information processing [22].

## REFERENCES

[1] R. Lynch "The quantum phase problem", Phys. Rev., vol. 256, issue. 6, pp-367-436, 1995.
[2] Z. Ficek, M.R. Wahiddin "Quantum Optics Fundamental and applications", IIUM, Kuala Lumppur, 2004(chapter 5).
[3] V.Perinova, A. Luks, J. perina "Phase in Optics", World Scientific, Singapore, 1998(Chapter4).
[4] L. Susskind, J. Glogower "Quantum mechanical phase and time operator", Physics vol.1, issue. 1,pp. 49-61, 1964.
[5] S.M. Barnett, D.T. Pegg "Phase in quantum optics", J. Phys. A: Math. Gen., vol. 19, issue 18,pp.3849-3862,1986.
[6] D.T. Pegg, S.M. Barnett "Phase properties of quantized single mode electromagnetic field" Phys Rev. A , vol. 39, issue. 4, pp.1665-1675, 1989.
[7] H.Y. Fan, H.R. Zaidi "An exact calculation of the expectation values of phase operators in squeezed light", Opt. Commun., vol. 68, issue. 2, pp.143-148, 1988.
[8] B.C. Sanders, S. M. Barnett, P. L. Knight "Geometric phase distribution for open quantum systems", Opt. Commun., vol.58, pp.290, 1986.
[9] D. Yao "Phase properties of squeezed state of light", Phys. Lett. A, vol. 122, issue. 2, pp.77-83, 1987.
[10] C.C. Gerry "Quantum phase properties of non linear optical phenomenon" Opt. Commun.,vol. 63,pp.278, 1987.
[11] P.Carruthers, M. M. Nieto "Phase and angle variables in quantum mechanism" Rev. Mod. Phys,vol. 40, issue. 2, pp. 411-440, 1968.
[12] R. Lynch"Phase fluctuation in coherent light/anharmonic oscillator system via measured phase operator", Opt. Commun., vol. 67, issue. 1, pp.67-70, 1988.
[13] R. Lynch "Phase fluctuations in a squeezed state using measured phase operators" Opt. Soc. Am. B, vol. 4, issue 10, pp. 1723-1726, 1987.
[14] J. A. Vaccaro J. A. Vaccaro, D. T. Pegg "Experimental determination of number phase uncertainty relations" Opt. commun., vol. 105,pp.335, 1995.
[15] Y. K. Tsui, M. F. Reid "Unitary and Hermition phase properties for electromagnetic field", Phys. Rev. A, vol. 46, issue. 1,pp.549-554, 1992.
[16] A. Pathak, S. Mandal "Phase fluctuation of coherent light coupled to a nonlinear medium of inversion symmetry", Phys. Lett. A,vol. 272, issue. 5-6, pp. 346-352, 2000.
[17] A.B. Klimov, L.L. Sanchez-Soto, H.D.Guise and G. Bjork "Quantum phases of a qutrit", J. Phys. A:Math. Gen., , vol. 37,issue 13, pp. 4097-4106, 2004.
[18] A. Luis, L.L. Sanchez Soto "Experimental demonstration of phase difference operator", Phys. Rev. A, vol. 48,issue. 6, Pp. 4702-4708, 1993.
[19] I. Iguchi, T. Yamaguchi and A. Sugimota "Diamagnetic activity above $T_C$ as a precursor to superconductivity in $La_{2-x}Sr_xCuo_4$ thin films", Nature, vol. 412, issue.4862,pp. 420-423, 2001.
[20] A Zavatta, S Viciani, M. Bellini "Quantum to classical transition with single photon added coherent states of light", Science, vol. 306 , Issue 5696, pp. 660-662, 2004.
[21] A. Pathak, P. Gupta "Reduction of quantum phase fluctuations means antibunching", Physics Letters A, vol. 365, pp. 393-399, 2007.
[22] A. Beveratos, R. Brouri, T. Gacoin, A. Villing, J.P. Poizat and P. Grangier "Single photon quantum cryptography", Phys. Rev. Lett., vol. 89, issue 18, pp. 187901-187904, 2002

### Authors Profile

Dr. Savita Gill pursued M.sc in Physics from Kurukshetra University in 1997 and topped the University (gold medalist). She did M.Tech in Instrumentation from NIT Kurukshetra with distinction in 2000 and Ph.D in Physics from Kurukshetra University in 2012. She is currently working as Assistant Professor in University Institute of Engineering &Technology, Kurukshetra University since 2006. She has published more than 20 research papers in reputed international journals including SCI. Her specialization research work focuses on Nonlinear Optics and Quantum Optics. She has 13 years of teaching and research experience.

 