International Journal of Scientific Research in Physics and Applied Sciences
Research Paper
Vol.8, Issue.4, pp.27-34, August (2020)
E-ISSN: 2348-3423

Revolutionary Motion of a Body under the Action of Resolution of Acceleration Due To Gravity Involved

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Available online at: www.isroset.org

Received: 30/Apr/2020, Accepted: 05/Jun/2020, Online: 31/Aug/2020

Abstract- The revolutionary motion of a body under the force of gravitation is treated in view of its motion in a two dimensional plane. The change in the direction of velocity of the revolving body for going into a circular orbit can be understood on the basis of gravitational force appearing during the motion to act for showing its effect two dimensionally. Therefore, the resolution of force is undertaken to exist along two mutually perpendicular directions. Because, the force causes acceleration to act on velocity. Hence, the horizontal and vertical components of acceleration due to gravity play a dominant role to change the corresponding components of velocity. Newton's law of motion with regard to the mechanics of accelerating and retarding phenomena is applied by the way. The investigated components of acceleration due to the gravity vary as according to $aH = g\ \text{Sin}\Delta\theta$ and $aV = g\ \text{Cos}\Delta\theta$, where $\Delta\theta$ is the measurement of angular displacement of the body during the motion from its initial position. The resultant of these two components is related by $g = V^2 / R$ with the orbital velocity for providing requisite centripetal force equal to the gravitational force acting towards the centre of the orbit. Because, the involved centripetal acceleration acts normally to orbital velocity V. Therefore, such an acceleration is called a revolutionary normal acceleration. The changing magnitudes of the horizontal and vertical components of velocity is found to be the actual cause for changing the direction of resultant velocity of the body for remaining ever acting towards the orbit during the motion. The revolutionary motion in an elliptical orbit is also tried to understand by the application of such components involved during the motion under the changing magnitude of gravitational force acting towards the focus of the ellipse. The whole motion in an elliptical orbit is simplified by correlating the observations for obtaining the motion in the trajectory of two successive conjugate parabolic envelopes of the same shape.

Keywords- Gravitational force, centripetal force, revolutionary motion, circular motion, elliptical motion, orbital velocity, acceleration due to gravity, resolution, acceleration & retardation, parabolic motion, enevelopes, perigee & apogee, focus.

I. INTRODUCTION

When a body revolves around a point in a circular orbit, a force is expected to act on the body towards the centre of the circle. The force may be of any kind viz. gravitational, electrostatic or magnetic. In case of revolutionary motion of bodies under the gravitational force, Newton's universal gravitational law applies. According to it, an universal force of attraction acts in between the masses of two bodies. The force is so called the universal gravitational force. Because, the force acts through the line joining the centre of gravities [1]. The planets are the main example of such a motion around the great mass of the sun. Before going in detail to know about the type of planetary motion, the basic theory should be focused firstly upon the study of circular motion of a revolutionary body around a point for which a centripetal force equal to $(m V^2 / R)$ is necessary. During such motion around a circular path, the body which moves around is found to have an orbital velocity V. However, the magnitude of this orbital velocity remains unchanged, but due to the change in its direction by vectorial distinction, the orbital velocity can be treated as changing throughout the whole motion around the orbit. Thus, when a body revolves around a circular orbit, its velocity goes on changing due to the vectorial distinction. This change is described by creating its motion in a two dimensional plane. The change in velocity components towards two mutually perpendicular directions occur due to the change of force acting along these directions. This can be possible only due to the acting of resolved components of the force. As a result of this type of change, the involvement of resolution of acceleration due to gravity associated with the motion also becomes very significant. Although, the acceleration due to gravity at the initial point of consideration seems to have only one component owing to the cause of its acting towards perpendicularly to the orbital velocity. But, both the components of acceleration begin to appear as the motion proceeds forward due to the change of direction of acceleration. In the same way, at the initial point of consideration, the orbital velocity also seems to have only one component owing to the cause of its acting towards perpendicularly to the acceleration. But, both the components of velocity begin to appear due to the change of direction of velocity as the motion proceeds forward.

Because, the direction of orbital velocity goes to change at every moment during the motion. Due to the change of direction of force, the horizontal and vertical components of velocities will also go to change from its initial values along the same directions formatting the plane used for the treatment of the motion. To find out the cause for it, the introduction of horizontal and vertical components of acceleration is found equally important. Thus, the horizontal and vertical components of acceleration are assumed to be the responsible for the change of velocity components along those directions. Resolved components of velocity are seen to change during the motion so far considered between the two points. At the initial reference point, the direction of orbital velocity is chosen as the X axis for obtaining the horizontal component during the motion and the perpendicular to it is selected as the Y axis for getting the vertical component towards it. The resolution is done to assess both types of the changes occurring in velocity as well as in acceleration responsible behind the motion.

The revolutionary motion is entirely different from the linear type of motion. Because in such a motion, the force is not applied linearly along the direction of the velocity. But, the gravitation force is acted normally to the velocity. Such a force can not take the body for a simple type of linear motion. The force carries the body in a two dimensional circular type of motion. Therefore, the effect of gravitational force acting on the velocity can be understood on the basis of the effect of its components acting on the respective components of velocity suitably available in two mutually perpendicular directions. How much a component of velocity will be changed in its own fixed direction by the corresponding component of force acting along that direction, it all depends upon the resolved component of acceleration due to gravity aligned towards the respective fixed direction. The changing direction of force during motion will apply changing components of acceleration to act on velocity components of the body for varying magnitudes of components with respect to each other. Thus, Newton's law of motion is found to be applicable to derive the theory by accelerating and retarding processes occurring along two mutually different linear directions possessing the velocity components responsible for circular type of whole motion.

II. THEORY

When a body moves from one point to another in any direction under a force to act on it, its velocity goes to change. If the force acts opposite to the direction of motion, the velocity is retarded by the force. Because, the acceleration acts always in the direction of force, it will be also opposite to the direction of motion. Such an acceleration causes to change the velocity, hence is responsible for decreasing the velocity component from its initial value. But, if the force acting on the body is applied towards the same direction of motion, the velocity is accelerated by the force. Because, the acceleration acts always in the direction of force, it will be also in the same direction of motion. Such an acceleration causes to change the velocity, hence is responsible for increasing the velocity component from its initial value.

III. MECHANISM

Let a body of mass m is moving in a circular path with an orbital velocity V around a fixed mass M. The gravitational force which the mass M exerts on the moving body is (m g), where g is the acceleration due to gravity at the distance of the orbit from its centre. Now, let us treat the momentum, its direction and change of its rate with the time. Suppose the body is initially at A, where the velocity vector is VA. During motion, the velocity vector at B is VB. Let the body takes time Δt in reaching from A to B by an angular displacement of $\Delta\theta$. During this time, the orbital velocity changes from VA to VB along directions as shown in figure 1. According to Newtonian classical mechanics, the rate of change in momentum is equal to the force applied [2]. Hence, the second law of motion relating the force and acceleration is also applicable. We define the linear motion in the following way.

$m\ a = F = \Delta p / \Delta t$

We should now take this method of representation of expressions into account. The two components of velocity are available at each point between which the motion is considered. When the body goes from one position of A to another position at B, the force acting on the body at the final position at B is analyzed and is found to have changed the velocity components along horizontal and vertical directions. Hence, the resolution of force at its final position is also taken along horizontal and vertical directions. Newtonian mechanics during motion from A to B can be used as according to -

Applying horizontal components of force, velocity and resolved component of acceleration aH towards it.

For a motion of $VB\ Cos\Delta\theta < VA$

$$m\ aH = F\ Sin\Delta\theta = (m\ VB\ Cos\Delta\theta - m\ VA) / \Delta t$$
$$= F\ Sin\Delta\theta$$
$$= m\ g\ Sin\Delta\theta$$
$$aH = g\ Sin\Delta\theta$$

Applying vertical components of force, velocity and resolved component of acceleration aV towards it.

For a motion of $VB\ Sin\Delta\theta > 0$

$$m\ aV = F\ Cos\Delta\theta = (m\ VB\ Sin\Delta\theta - 0) / \Delta t$$
$$= F\ Cos\Delta\theta$$
$$= m\ g\ Cos\Delta\theta$$
$$aV = g\ Cos\Delta\theta$$

Thus, the horizontal and vertical components of acceleration acting on the revolutionary body depends upon the angle of displacement $\Delta\theta$ and the expressions for these are measured as $aH = g\ Sin\Delta\theta$ and $aV = g\ Cos\Delta\theta$. The resultant of these two vectors can be calculated as –

$a = \sqrt{[(aH)^2 + (aV)^2]}$
$= \sqrt{[(g\, Sin\Delta\theta)^2 + (g\, Cos\Delta\theta)^2]}$
$= \sqrt{[g^2 (Sin^2\Delta\theta + Cos^2\Delta\theta)]}$
$= g$

Because, the resultant acceleration is equal to the acceleration due to gravity which acts ever normal to the direction of orbital velocity. Hence, such an acceleration is also called the revolutionary normal acceleration. Therefore, by changing the notation of indication, we can write as-

$aN=g$
When $\Delta\theta = 0$, $aH = g\, Sin\Delta\theta = g\, Sin0 = 0$
$aV = g\, Cos\Delta\theta = g\, Cos0 = g = aN$
When $\Delta\theta = 90$, $aV = g\, Cos\Delta\theta = g\, Cos90 = 0$
$aH = g\, Sin\Delta\theta = g\, Sin90 = g = aN$

It can be manipulated from these data that what ever be the position of revolutionary body in its orbit, an acceleration acts on it normal to its position of velocity vector. Although, the acceleration due to gravity acts it self as a centripetal acceleration required necessarily for the revolutionary motion of a body but it has a relation with orbital velocity V which can be obtained with the help of the following equation through the various mathematical steps as according to need.

$m\, aV = (m\, VB\, Sin\Delta\theta - 0) / \Delta t$
$aV = VB\, Sin\Delta\theta / \Delta t$
As $\Delta\theta \rightarrow 0$, $Sin\Delta\theta \rightarrow \Delta\theta$, $|VB| = |VA| = V$ and $aV = aN = g$ $g = V\, \Delta\theta / \Delta t$ but $\Delta\theta = arc / radius$
$= V\, AB / R\, \Delta t$, where $AB = arc$
$= V\, \Delta s / R\, \Delta t$, where Δs = linear displacement
$= V / R\, \{\Delta s / \Delta t\}$ but $(\Delta s / \Delta t) = V$
$= V^2 / R$

This is the relation between the acceleration due to gravity and the centripetal acceleration. Thus, if a body is required to revolve around a circular orbit of radius R and by an orbital velocity V, the centripetal acceleration depending upon its velocity represented by the expression (V^2 / R) must be exactly equal to the value of acceleration due to gravity g at that height of the orbit around a fixed mass M. Now, we will also find the relation between the centripetal force and the gravitational force involved during such type of revolutionary motion by using well known concept of change of acceleration due to gravity with the height by putting ($g = G\, M / R^2$).

$V^2 / R = g$ (Multiplying both the sides by m)
$m\, V^2 / R = m\, g$
$m\, V^2 / R = m\, G\, M / R^2$
$m\, V^2 / R = G\, M\, m / R^2$

In this way, the centripetal force remains equal to the Newton's gravitational force when a body is found to revolve under the force of gravitation [3]. Thus, for the revolutionary motion of a body –

$m\, V^2 / R = G\, M\, m / R^2$
$V^2 = G\, M / R$
$(2\, \pi\, R / T)^2 = G\, M / R$
$4\, \pi^2\, R^2 / T^2 = G\, M / R$
$T^2 = (4\, \pi^2 / G\, M)\, R^3$

In the universe, many bodies are found to revolve around the gravity of other bodies respectively. In such cases, the period of revolution depends upon the mass M situated at the centre as well as upon the radius of the orbit. Therefore, the formula for the period of revolution must be rearranged in the context of universe according as-

$T^2 = (4\, \pi^2 / G\, M)\, R^3$
$= (R^3 / M)\, (4\, \pi^2 / G)$
$= I\, R^3 / M$
where $I = (4\pi^2 / G) = 5.91 \times 10^{11}\ kg^2 / N\, m^2$

Thus, what ever be the mass m of a revolving body, if it is revolving in the circular orbit of radius R around a fixed mass M, its interval of revolution is fixed by this formula.

IV. RESULTS AND DISCUSSUION

[A] CIRCULAR TYPE OF REVOLUTIONARY MOTION

It is clear from the revolutionary motion of a body around a circular path under the gravitational force of a fixed mass M, that a body moving with a certain velocity V can only revolve in a circular path in the presence of acceleration arising due to the field of gravity. The force of gravitation shows its effect to change the direction of the moving body regularly for keeping in revolutionary motion in a circular orbit of radius R. The motion for revolution is only possible if the body has such a magnitude of velocity for which
$V = \sqrt{(G\, M / R)}$. The change in direction of gravitational force allows circular motion to be explained by considering the whole motion in a two dimensional plane. Therefore, the resolution of force is treated by a simple rule of resolving the components along two mutually perpendicular directions. Because, the acceleration due to gravity is directly related with the gravitational force. Hence, the two dimensional horizontal and vertical components of acceleration also exist. These components will help to find out the magnitudes of respective components of orbital velocity during the motion. The resolution of acceleration due to gravity is accompanied by the effect of its components on the respective components of velocity of the moving body. The finding of magnitudes of the components of velocity decides the actual direction provided to the resultant velocity for the motion to proceed. An X-Y type of plane is used to design the pattern for getting directions for the components. The change in direction of a component is an indication of forwarding the motion to proceed towards the prescribed orbit. When the horizontal component of acceleration is maximum during the motion, the vertical component of acceleration is minimum. And, when the horizontal component of acceleration is minimum during the motion, the vertical component of acceleration is maximum. It is due to the cause of depending of both the components according to $aH = g\, Sin\Delta\theta$ and $aV = g\, Cos\Delta\theta$, where $\Delta\theta$ is the displacement during the motion of the body from its initial

position of consideration for the treatment of motion two dimensionally. The resultant of aH and aV remains always acting by an unchanged magnitude of g towards the centre of the circle. This happens due to the unchanged magnitude of gravitational force acting on the body towards the centre of the orbit. Because, the distance of the greater mass from the revolving body is the measurement of the gravitational force acting during the motion. Therefore, due to remaining of equal distance of revolving body from the centre of circular orbit, the magnitude of gravitational force acting on the body remains invariable through out the whole motion.

The whole motion around the circle can be divided into four equal parts of path length with the help of two mutually perpendicular lines known as horizontal and vertical diameters of the circle. When the horizontal component of velocity is retarded by the horizontal component of acceleration in any part, the vertical component of velocity is being accelerated by the vertical component of acceleration in the same part. Then, during the motion in the next part, the horizontal component of velocity is accelerated by the horizontal component of acceleration and the vertical component of velocity is retarded by the respective vertical component of acceleration. The same pattern of change repeats in the next two parts also. Thus, during the motion of a body in its circular orbit, when the horizontal component of velocity goes on decreasing, the vertical component of velocity goes on increasing and when the horizontal component of velocity goes on increasing, the vertical component of velocity goes on decreasing. Due to the effect of resolved components of acceleration due to gravity, when the horizontal component of velocity attains its maximum value V, the vertical component of velocity gets a minimum value zero and when the vertical component of velocity attains its maximum value V, the horizontal component of velocity gets a minimum value zero. At the vertexes of vertical diameter when the acceleration due to gravity acts vertically towards the centre of the circle, the vertical component of velocity is zero. Similarly, due to passing through the vertexes of horizontal diameter, when the acceleration due to gravity acts horizontally towards the centre of the circle, the horizontal component of velocity is zero. But, as a result of this type of whole change, the resultant of both the components remains always acting towards the tangent of the circular orbit by an unchanged magnitude of orbital velocity V and the direction of velocity remains always normal to the gravitational force for a body moving in the orbit of a circle.

It can be seen that during the motion from the point A to the point B in figure 1, that the direction of orbital velocity has been changed by an angle $\Delta\theta$. The horizontal component of velocity has become VB Cos$\Delta\theta$ due to retardation by horizontal component of acceleration aH. The vertical component of velocity gets a value VB Sin$\Delta\theta$ due to the acceleration by vertical component of acceleration aV. It should be also noticed that when the body was initially at the point A, the horizontal and the vertical components of velocity were VA and 0 respectively. The resultant of horizontal and vertical components of velocities can be obtained accordingly.

The resultant orbital velocity at the point A = |VA| = V and at B = $\sqrt{[(VB\ Cos\Delta\theta)^2 + (VB\ Sin\Delta\theta)^2]}$
$= \sqrt{[VB^2 (Cos^2\Delta\theta + Sin^2\Delta\theta)]} = |VB| = V$

It is quite obvious that the components of orbit velocity are both related with the change of direction of orbital velocity by an angle $\Delta\theta$. Therefore, the finding of changing magnitudes of components of velocity is responsible for changing the direction of orbital velocity during the motion. But, the finding of resultant orbital velocity is independent of $\Delta\theta$. Therefore, the total magnitude of orbital velocity remains unchanged during the whole motion in a circular orbit. This is because of |VA| = |VB| = V. Thus, the main objective of the investigation carried on circular type of revolutionary motion with an aim to know about the cause for the change in direction of the orbital velocity is successfully achieved by treating the motion two dimensionally with the help of resolution of acceleration due to gravity. These are responsible for the change in respective magnitudes of components of velocity to give the actual direction to the resultant velocity during the revolutionary motion in the circular orbit.

[B] ELLIPTICAL TYPE OF REVOLUTIONARY MOTION

In an elliptical type of revolutionary motion, the role of the velocity components becomes important under the changing magnitude and direction of a gravitational force and the related acceleration acting with respect to the position of the body in motion. Because in such a motion, the body does not move with a constant magnitude of velocity. If the attained velocity takes the body for an elliptical type of orbital motion around the greater mass kept at one of the foci, the distance of the revolving body will change from the focus during this motion. The magnitude and the direction of gravitational force also changes accordingly. The change in velocity from one point to another during the motion in an elliptical orbit is assisted by the change of magnitude as well as by the direction of force due to the change of distance. The changing force applies a change in acceleration and a changing acceleration is responsible for the change in velocity. The changing behavior of components of gravitational force acting on the body during motion in an elliptical orbit is demonstrated in a two dimensional plane in figure 2. The two dimensional resolution of the force is responsible for applying respective components of acceleration to act for changing the corresponding components of velocity for the orbit.

The elliptical motion of a body appears to occur generally around the centre O of the ellipse by the periphery of vertexes of longer and shorter axis of the ellipse. Due to the presence of greater mass put at the focus, the gravitational force exerted by it on the revolving body towards the focus is dominant to make the motion to occur very symmetrically as required for the orbit of the ellipse. When the horizontal component of velocity keeps the body to move outside horizontally, the vertical component of velocity acts to carry the body inside vertically. Similarly, when the vertical component of velocity keeps the body to move outside vertically, the horizontal component of velocity acts to carry the body inside horizontally. Thus, the role of resolved components of acceleration acting on respective components of velocity becomes important for understanding the mechanism of planetary type of elliptical motion. The simplified observations in this regard are co related with the theory developed. According to this concept, the component of velocity of the body in motion is retarded, if it is directed oppositely to the respective component of acceleration acting on it. The component of velocity of the same body is accelerated in motion by the corresponding component of acceleration, if it is directed towards the same.

For determining the effect of horizontal component of acceleration on its respective horizontal component of velocity during motion in an elliptical orbit, the role of shorter axis becomes important. Because, the whole motion around the ellipse is needed to divide into four equal parts of path length with the help of two mutually perpendicular lines known as longer axis and shorter axis of the ellipse. If, the horizontal component of velocity is retarded by the horizontal component of acceleration in any part, then during the motion in the next part, the horizontal component of velocity is accelerated by the horizontal component of acceleration acting during the motion. The same pattern of change repeats in the next two parts also. Thus, the horizontal component of velocity is retarded in two parts during motions from A to D and from E to F. While, the horizontal component of velocity is accelerated in two other parts during motions from D to E and from F to A respectively.

Similarly, for evaluating the effect of vertical component of acceleration on its respective vertical component of velocity during motion in an elliptical orbit, the role of latus rectum becomes important due to the presence of the source of gravitation at the focus. The whole motion around the ellipse is needed to divide into four different parts of path length with the help of two mutually perpendicular lines known as longer axis and the latus rectum of the ellipse. If, the vertical component of velocity is accelerated by the vertical component of acceleration in any part, then during the motion in the next part, the vertical component of velocity is retarded by the vertical component of acceleration acting during the motion. The same pattern of change repeats in the next two remaining other parts also. Thus, the vertical component of velocity is accelerated in two parts during motion from A to C and from E to G. While, the vertical component of velocity is retarded in two other parts during motion from C to E and from G to A respectively.

Now, the overall observations can be used to interpret the elliptical type of revolutionary motion very clearly. As soon as the body proceeds forward from the reference point A with velocity VA towards the tangent of the orbit of ellipse, the resolved components of acceleration aH = g $\text{Sin}\Delta\theta$ and aV = g $\text{Cos}\Delta\theta$ appear to act with varying magnitudes due to the change occurring in both angular displacement $\Delta\theta$ and acceleration due to gravity g by motion. The horizontal component of velocity is retarded by aH from its initial value VA to 0 on reaching from the point A to D. While in this duration, the vertical component of velocity is accelerated drastically by aV from its 0 initial value during motion from the point A up to C of the vertex of latus rectum and then goes to retard due to change in direction of aV. At the vertex of shorter axis at D, the vertical component of velocity is retarded to get a value VD. This retarded vertical component of velocity VD becomes as the resultant orbital velocity at D. Because, the horizontal component of velocity is 0 at D. As the body moves from the point D with velocity VD towards the apogee of the ellipse, the horizontal component of velocity is now accelerated from 0 to VE on reaching from the point D to E. While, the vertical component of velocity still remains retarding from its value VD to 0 on reaching from the point D to E. Thus at the vertex of longer axis at E, the accelerated horizontal component of velocity VE acts as the resultant orbital velocity. Because, the vertical component of velocity is 0 at E. In the context of resultant orbital velocity, this velocity VE of the revolving body becomes the minimum attained velocity in the orbit of ellipse due to retardation as a result of climbing towards the peak of the ellipse by the motion against the gravitational field. As the body moves from the point E with velocity VE towards the orbit of the ellipse, the horizontal component of velocity is now retarded from VE to 0 on reaching from the point E to F. While in this duration, the vertical component of velocity is accelerated from its 0 initial value to VF on reaching from E to F. At the vertex of shorter axis at F, the accelerated vertical component of velocity VF acts as the resultant orbital velocity. Because, the horizontal component of velocity is 0 at F. As the body moves from the point F with velocity VF towards the perigee of the ellipse, the horizontal component of velocity is now accelerated from its initial value 0 to VA on reaching from the point F to A. While in this duration, the vertical component of velocity still remains accelerating during motion from the point F up to G of the vertex of latus rectum and is then retarded to get a value 0 during motion from G to A. Thus, at the vertex of longer axis at A, the accelerated horizontal component of velocity VA acts as the resultant orbital velocity. Because, the vertical component of velocity is 0 at A. In the context of resultant orbital velocity, this velocity VA of the revolving body becomes the maximum attained velocity in the orbit of ellipse due to acceleration as a result of down fall towards the bottom of the ellipse by motion towards the direction of gravitational field.

In case of elliptical motion, the resultant of aH and aV remains always acting by a changing magnitude of g towards the focus of the ellipse and the resultant of both the velocity components remains always acting towards the tangent of the ellipse by a changing magnitude of orbital velocity V. This happens due to the cause of change of the distance of the revolving body from the focus of the ellipse. The velocity of the body remains normal to the gravitational force only at two places when it passes through the vertexes of the longer axis of the ellipse. Due to consideration of motion in a two dimensional plane, at the vertexes of longer axis of the ellipse, the horizontal component of acceleration is zero. The total resultant velocity of the body at each of the vertex is only equal to the horizontal component of velocity alone. Because, the vertical component of velocity is zero at these points. On arriving at the vertex of perigee, the horizontal component of velocity attains its maximum value VA due to the involvement of highest magnitude of gravitational field. Therefore, during the whole motion, the orbital velocity of the revolving body remains highest at this vertex only. On reaching at the vertex of apogee, the resultant orbital velocity is again only equal to the horizontal component of velocity. But in this time, the attained velocity is low enough and is only VE due to the involvement of lowest magnitude of gravitational field. Therefore, during the whole motion, the orbital velocity of the revolving body remains lowest at this vertex only. Thus, the results obtained are in accordance to the study made by Kepler [4]. The magnitude of the orbital velocity does not remain the same at each point in case of elliptical orbit. When the distance of the body goes on decreasing from the focus, the resultant orbital velocity of the body goes on increasing due to gradual increasing rate of acceleration acting on it. But, when the distance of the body goes on increasing from the focus, the resultant orbital velocity of the body goes on decreasing due to the gradual decreasing rate of acceleration acting on it. At one of the vertex which is far from the focus, the velocity of the revolving body remains minimum. This is called the position of apogee of the elliptical orbit. While, at the other vertex which is nearest to the focus, the velocity of the body remains maximum. This is called the position of perigee of the elliptical orbit. Thus, the resultant orbital velocity is retarded during the motion in half of the elliptical path while, the resultant orbital velocity is accelerated during the motion in the next half path of the ellipse. The retardation is caused by the motion of the revolving body beyond the source of gravitational field due to increasing distance from the focus. While, the acceleration is caused by the motion of revolutionary body towards the source of gravitational field due to the decreasing distance from the focus. The value of gravitational field, hence the value of g acting on the revolving body changes with the distance of it from the source of gravitational field situated at the focus of the ellipse. The numerical value of g is highest, when the revolutionary body passes through the nearest. The value of g is lowest, when the revolving body passes through the farthest distance from the focus. The orbital velocity changes as according to the change of magnitude and direction of g acting for forwarding the motion. Because, the attaining of velocity is ruled by a relation with the magnitude and direction of acceleration acting during the motion of a body. As a result of this effect, when a body revolves under gravitation in an elliptical orbit, its velocity is fast, when it goes though a point which is nearest to the focus. The same body is slowed down mostly while passing through a point which is farthest from the focus.

At the vertexes of latus rectum, the acceleration due to gravity acts horizontally towards the focus of the ellipse. Therefore, the vertical component of acceleration is zero at these two points. The resultant orbital velocity is inclined by an angle $\Delta\Phi$ with the horizon. Because, $\Delta\Phi$ is the measurement of change of direction of the orbital velocity from its initial direction. The resolved components of velocity depend upon this angle $\Delta\Phi$ as shown at the various points A, B, C & D in figure 2. The change in magnitudes of the horizontal and vertical components of the velocity during motion in an elliptical orbit will be responsible for the change of direction of orbital velocity as well as for deciding the change in its magnitude during the revolution under the resolved components of changing magnitude of acceleration due to gravity.

At the vertexes of the shorter axis, the total resultant orbital velocity at these two points is equal to the vertical component of velocity alone. Because, the horizontal component of velocity is zero at each of these two points. The direction of orbital velocity is perpendicularly inclined ($\Delta\Phi = 90°$) with the horizon at these two points D and F. The direction of acceleration due to gravity of gD acting at D on VD is such that it projects the body to go for a locus of parabolic envelope DEF. Similarly on reaching at F, the direction of acceleration due to gravity of gF acting at F on VF is such that it projects the body to go for a locus of parabolic envelope FAD.

Thus, the whole motion in an elliptical type of orbit seems to be a result of motion of a body in two successive parabolic type of conjugate envelopes. These parabolic motions are situated across both the sides of the shorter axis of the ellipse. The orbital velocity attained at each corner of the shorter axis plays the role of projectile velocity for the beginning of each parabolic motion. One of the parabolic motion occurs towards the apogee of the ellipse. While, the other occurs towards the perigee of the ellipse. These two complete parabolic motions are performed in the presence of two entirely different kinds of alignment of acceleration due to the gravity. In one type of parabolic motion towards the peak of apogee of the ellipse, the resultant projectile velocity goes on decreasing due to the retardation by the climbing motion against the decreasing magnitude of acceleration acting due to the force of attraction. At the vertex, the horizontal component of the velocity is assisted by the perpendicularly applying acceleration due to gravity for forwarding the motion beyond. The downfall of the projectile parabolic motion takes place with increasing magnitude of velocity as a result of acceleration produced by the changing direction of increasing attractive force acting towards the movement. As the parabolic type of motion is reached at the depth of shorter axis in front, the parabolic motion of the first envelope is completed. Now, due to the free space available for motion, the attained velocity is taken over as the projectile velocity for the beginning of second type of parabolic motion to proceed towards the bottom of the perigee of the ellipse. But, this parabolic motion is performed inversely due to the presence of the acceleration due to gravity acting for the motion uniquely. Because in this case, the velocity during the motion towards the vertex of perigee of the ellipse is accelerated due to the acceleration caused by the force of attraction acting towards the movement. Hence, the magnitude of velocity increases during downfall motion towards the vertex of the perigee. At the vertex, the horizontal component of velocity is assisted by the perpendicularly applying acceleration due to gravity for forwarding the motion beyond. The climbing of projectile parabolic motion takes place with decreasing magnitude of velocity as a result of retardation produced by the changing direction of attractive force acting against the movement. As soon as, the climbing motion reaches at the height of shorter axis in front, this parabolic motion in the second envelope also gets completed. Thus for simplification, the elliptical type of revolutionary motion is bifurcated into two conjugate parabolic envelopes of the same shape.

[C] HOW TO PROBE INTO THE REVOLUTIONARY ORBIT

The orbital velocity in case of circular orbit can be determined by equating the relation ($g = V^2 / R$). Because, the distance of the revolving body and the acceleration due to gravity acting remains unchanged through out the whole motion. For a distance of revolution around the fixed mass kept at the centre of the circle, the necessarily required orbital velocity is related with the centripetal force and the centripetal force is related with the force of gravitation acting on the body at that height of the orbit from its centre. When a body is given an appropriate velocity for moving in a circular orbit and if the body goes to move in a circular orbit by this velocity under the influence of gravitational force, then the given velocity becomes the orbital velocity. But, the orbital velocity will be now called the resultant of its components. Because, the magnitude and direction of resultant velocity both are fixed by the acceleration due to gravity acting on the velocity during the motion two dimensionally.

In case of elliptical type of revolutionary motion, the orbital velocity can not be measured directly by applying such relations. Because, the distance of the body does not remain constant during the motion. The greater mass which exerts the gravitational force on the body is situated at the focus. Hence, the acceleration due to gravity acting on velocity of the body changes at every moment during the motion. So, an alternative method is adopted to find the orbital velocity for probing into the orbit of the ellipse. The mechanism is obtained as according to the fact.

Let, the ellipse selected has the focus S and is made by longer and shorter axes of lengths 2a and 2b respectively. Suppose, the shorter axis meets with the ellipse at the point D. The point D will be the vertex of the shorter axis. Because, the vertical component of velocity VD is perpendicularly inclined to the horizon at this point D. This velocity VD acting towards the tangent of the ellipse represents the value of orbital velocity required for going to ahead in the orbit of ellipse. For obtaining the calculations for this projectile velocity, the acceleration due to gravity gD acting on the body at D is undertaken to resolve into its components. The horizontal component of acceleration $aH = gD \, Sin\Delta\theta$ is found to act horizontally towards the centre of the ellipse. Now, if we draw a circle of radius OD= b around the centre of the ellipse O, the horizontal component of acceleration seems to act as if a centripetal acceleration is applying towards the centre O of this circle. Because, it acts normally to tangential velocity VD at the time when the retarded horizontal component of velocity is zero at D. Therefore, the involved centripetal acceleration can be used for making its statistical relation with the tangential velocity VD and the radius b of the uniform circle. For the mathematical treatment of a uniform circular motion, the necessary condition is that the centripetal acceleration formulated by the tangential orbital velocity must be basically equal to the cause of acceleration acting towards the centre of the orbit. Therefore, the tangential velocity VD positioned at the circular point D is chosen as the measurement of horizontal component of acceleration defined by the relation -

$$VD^2 / b = gD \, Sin\Delta\theta$$
$$VD^2 = b \, (gD \, Sin\Delta\theta)$$
$$VD = \sqrt{[b (gD \, Sin\Delta\theta)]}$$

This is the velocity required to probe into the orbit of the ellipse. The velocity is concerned as the projectile velocity needed for climbing towards the peak of apogee vertex of the ellipse from the vertex D of the shorter axis. The point D is situated at distance b apart from the centre O of the ellipse. The value of gD is the acceleration due to gravity at D acting towards the focus. While, $\Delta\theta$ is the angular displacement when measured at the focus during the motion considered from the vertex A up to the vertex D. However, the body must follow the orbital path due to the horizontal component of acceleration and due to velocity VD at D. But, owing to the presence of vertical component of acceleration, the resultant motion of the body is forwarded towards the orbit of the ellipse. This happens due to the acting of gD on VD at D towards the focus. Thus, the gravitational force acting towards the focus can take the body to move in an elliptical type of the orbit. If, the body is given an appropriate velocity required for probing into the orbit in view of direction of acceleration due to gravity acting two dimensionally.

V. CONCLUSION

Although, the revolutionary motion of a body under the gravitational force is well known in view of uniform circular motion, but in this investigation, it is interpreted by undertaking the motion in view of two dimensional plane. The change in direction of the revolving body can only be understood on the basis of the direction of force acting on the body for a two dimensional change. Therefore, the consideration of resolution of acceleration is applied in this theory. The effect of horizontal and vertical components of acceleration on respective components of velocity is assessed during the motion. The changing magnitudes of the horizontal and vertical components of velocity during the motion is found to decide the magnitude as well as direction for the resultant of orbital velocity. The circular motion, during which the velocity components are either being retarded or accelerated, is noticed to occur in the same four parts. This becomes plausible due to the equal amount of gravitational force acting on the orbital velocity for making the revolutionary motion very symmetrical as required for the circular type of orbit. But the elliptical motion, during which the velocity components are either being retarded or accelerated, is noticed to occur not in the same four parts but these are entirely different for each component. This happens due to the cause of changing magnitude of gravitational force imposed on velocity by the action towards the focus of the ellipse. Thus, the orbital velocity is assisted by the gravitational force for making the revolutionary motion very symmetrical as required for the elliptical type of the orbit. It is a matter of fact that during the motion in an elliptical type of orbit, the orbital velocity of the revolving body changes under the changing magnitude of acceleration. Because, the acceleration due to gravity depends upon the distance of the revolving body from the focus of the ellipse. The velocity increases during decrease of distance due to increasing magnitude of acceleration responsible for accelerating the velocity. But, the velocity decreases during increase of distance due to decreasing magnitude of acceleration responsible for retarding the velocity. Thus, both the velocity and the acceleration due to gravity at any point in the way of elliptical orbit depend upon the distance of the body from the mass M kept at the focus. It is also observed that for going to ahead into the orbit of ellipse, the body will have to follow the revolution by the locus in two successive conjugate parabolic envelopes of the same shape.

ACKNOWLEDGEMENT

The author likes to dedicate this effort in memory of his research guide Dr. J. C. Joshi and Dr. D. D. Pant for providing research facilities during luminescence investigation carried in past as a fellow of council of scientific and industrial research.

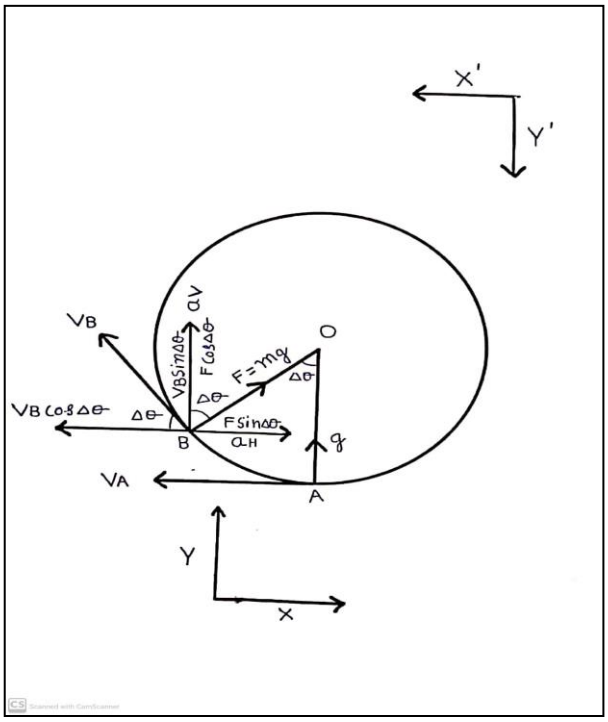

Figure 1. Demonstration of resolution of acceleration due to gravity during circular type of revolutionary motion.

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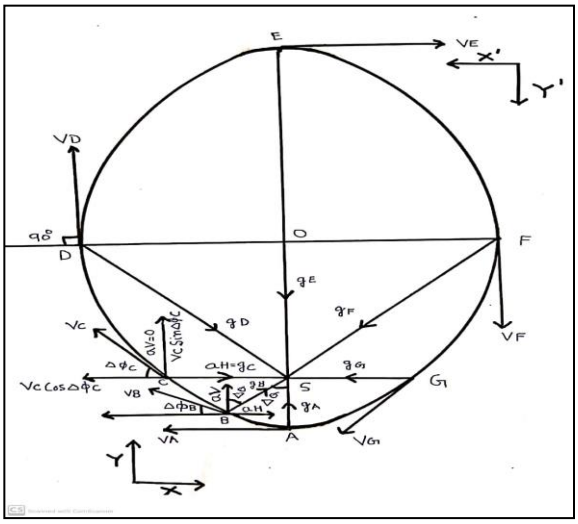

Figure 2. Demonstration of resolution of acceleration due to gravity during elliptical type of revolutionary motion

