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Dark Matter as Point-like Singularities: Alternative to Dark Halo Model

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Abstract**—**Observed galactic rotational curves have led to the inference of dark matter. The dark matter halo model is the most prevalent model to this day. This paper introduces an alternative model where dark matter is instead concentrated into massive singularities within galactic clusters. Three different hypothetical cases using this model will be presented here which will simulate rotational curves reminiscent of what has been observed in the study of dark matter. In these different cases, three different positions of a galactic plane orbiting a DM singularity will be modeled to show how the DM has influence on the rotation curves from when the DM singularity is over the center of the galactic plane, to the apsis, and consequently to any position in the orbit. This paper serves as an introduction to this model and a possible new direction in the study of dark matter.

Keywords—Dark Matter, Dark Matter Halo, Dark Halo Model Alternative**,** Dark Matter Singularity

I. INTRODUCTION

Observed galactic rotation curves such as in Figure 1 [1] where the orbital speeds of stars vs the radial distance from the galactic center have led to the postulation of dark matter, since the mass of the observable matter cannot fully explain these curves. The dark halo model, where dark matter is hypothesized as a diffuese distribution of matter permeating in and throughout galaxies and galactic clusters, is the most prevalent model to this day. This paper introduces an alternative theory to this model where dark matter is instead concentrated into massive singularities within galactic clusters. We will see in three instances within a simple system of a galaxy orbiting a DM singularity how easily we can mathematically model a rotation curve similar to what we see in Figure 1 [1]. This paper serves as an introduction to this model and how it can influence the future study of dark matter and an alternate option of how to move forward in this field of research.

The rest of the paper is organized as follows, Sections II and III contains assumptions of the nature of the dark matter singularities, and how it would be distributed within a galactic cluster, Sections IV, V, and VI mathematically models three different positions of a galaxy orbiting a dark matter singularity where the rotation curves will be calculated, Section VII concludes the work and provides the future scope of this research.

II. ASSUMPTIONS

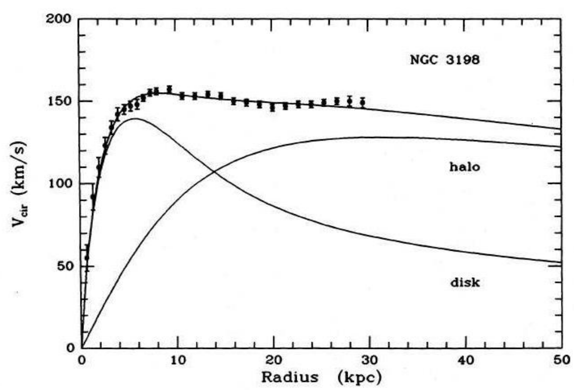

Figure 1. The rotation curve of spiral galaxy NGC 3198 [1]

I will not make many assumptions on the nature of the dark matter singularities themselves as this is not the purpose for this paper. An obvious assumption would be that they are obviously very massive and only emitting a strong gravitational force, which could explain why particles that have been hypothesized to be the composition of dark matter cannot be detected and may not even exist outside these singularities. But this does not assume that these singularities do not contain some sort of exotic particles themselves that may distinguish itself from black holes. Or whether these singularities are of a different nature or structure. And the final assumption I would make would be that these dark matter singularities were not created from ordinary, galactic matter like black holes are, but rather that they were conceived at the beginning or shortly after the big bang.

III. DARK SINGULARITY MODEL

Figure 2 roughly shows how the dark matter singularities might be distributed inside a galaxy cluster. Rather than being spread out as a cloud of particles, the DM (dark matter) is concentrated into point-like areas where galaxies orbit them as well as each other.

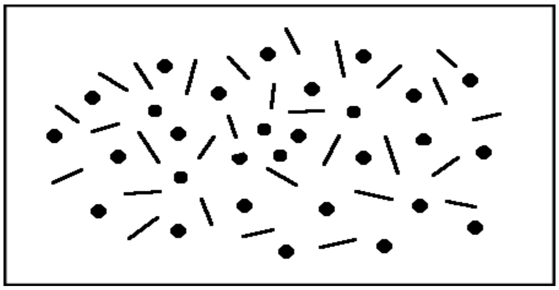

Figure 2. Galaxy Cluster showing lines as galaxies and dots as dark matter singularities

The singularities themselves may move around each other and may have a higher concentration of dark matter in the center of the cluster which may explain how x-ray emitting gas within clusters is brightest at the center as well as why the largest galaxy, such as in the Abell 2029 cluster, exist in this region, as more DM should give rise to bigger structures during the epoch of galaxy formation [2]. Figure 3 shows how light may still permeate through an ensemble of DM singularities and how gravitational lensing is viewed and mostly unchanged under such a model.

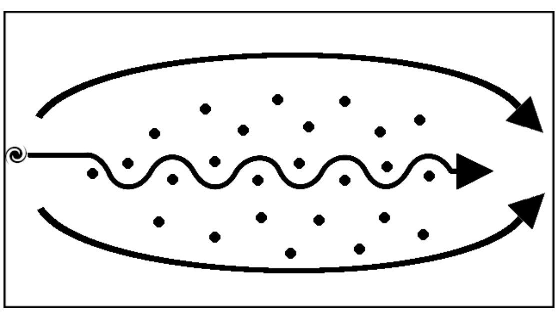

Figure 3. How gravitational lensing looks under Dark Singularity Model

IV. MODELING THE ROTATION CURVES

In the next three sections we will model the galaxy rotation curves for a simple system of a DM singularity and a galactic plane such as in figure 4. We will look at two other cases where the galaxy has shifted due to its orbit around the singularity. Due to distances involved any effects of General Relativity would be minimal to nil so we will only be using Newtonian gravity. Note that the galaxy is orbiting the DM singularity, so any movement towards the DM singularity is eliminated by the centrifugal forces.

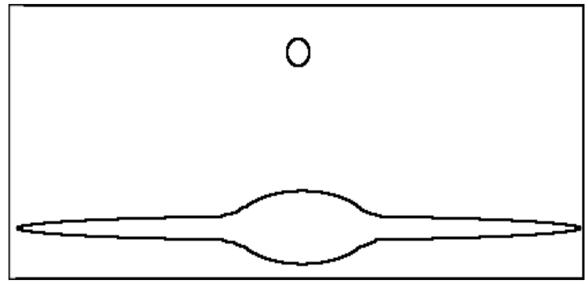

Figure 4. Simple case of DM singularity over center of a galaxy

To illustrate how the influence of the DM singularity will produce the rotation curves I'll simply show the components of the forces across the galaxy in Figure 5.

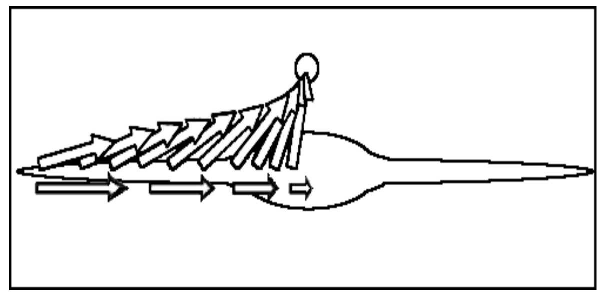

Figure 5. The components of the force as well as the resultant x-components

From the image you can see how the x-component of the forces across the galaxy will vary with distance. Figure 6 shows the diagram from which we will calculate the DM contribution of the rotation curve.

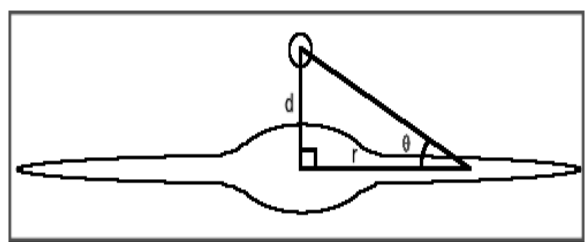

Figure 6. Diagram for the DM contribution where d is the distance of the DM singularity from the galactic plane and r is the varying distance from the center to a point on the galactic plane which we will plot the velocity vs. the radial distance (r)

$$a = \frac{GM}{r^2} \tag{1}$$

$$a = \frac{v^2}{r} \tag{2}$$

$$v = \sqrt{\frac{GM}{r}} \tag{3}$$

With the basic equations of gravitational and centripetal acceleration of (1) and (2) to produce (3) will be used to generate the rotation curves. I will make a few generalizations and definitions before continuing.

We will set G = 1 and M = 1 for convenience. We will do this to simplify the x-y plotting while making sure everything is in the right proportion more or less, eliminating all units. M_{LM} will represent the visible (light) mass of the galaxy, and M_{DM} for the mass of the dark matter singularity. We will have a ratio for the mass of M_{DM} and M_{LM} as being 8 to 1, so M_{LM} = ¼ M and M_{DM} = 2M, making the entire system composed of about 88%.dark matter and 12% visible matter.

To emulate the rotation curve of just the visible matter of a galactic disc, a factor of $e^{-\frac{1}{2\pi r}}$ will be inputted into equation (4) to produce (5) keeping in mind G = M = 1 and M_{LM} = ¼ M .

$$v=\sqrt{\frac{G(\frac{1}{4}M)}{r}} \tag{4}$$

$$v=\sqrt{e^{-\frac{1}{2\pi r}}\frac{G(\frac{1}{4}M)}{r}}$$

$$v=\sqrt{e^{-\frac{1}{2\pi r}}\frac{1}{4r}} \tag{5}$$

Figure 7 shows the result:

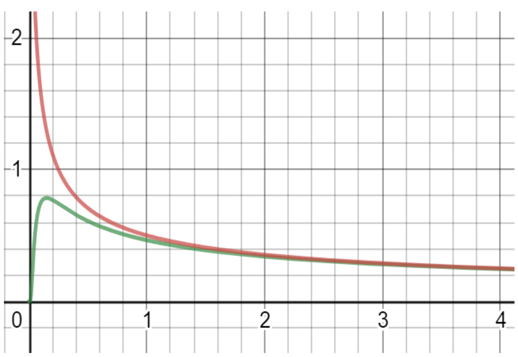

Figure 7. Plot of equations (4) in red and (5) in green.

For the DM contribution we will set up the equation using the diagram of Figure 6. In this scenario the galaxy is moving completely in the x direction as it orbits the DM singularity, so the centrifugal forces from the orbit won't affect the x-component of the force. So keeping in mind M_{DM} = 2M we have:

$$a=F/m=\frac{G(2M)}{r^2+d^2} \tag{6}$$

$$F_x = F\cos\theta \tag{7}$$

$$\theta = \arctan(d/r) \tag{8}$$

and using equation (2) and noting that we're only using the x-component of the acceleration gives us:

$$v=\sqrt{\frac{2GMr}{r^2+d^2}\cos(\arctan\frac{d}{r})} \tag{9}$$

and with G=M=1 and giving d, which is the distance of the DM singularity over the galactic plane, which we'll give a value of 2 (d=2) gives us:

$$v=\sqrt{\frac{2r}{r^2+d^2}\cos(\arctan\frac{2}{r})} \tag{10}$$

Figure 8 shows the plot of the DM contribution to the rotation curve which is similar to the one shown in Figure 1:

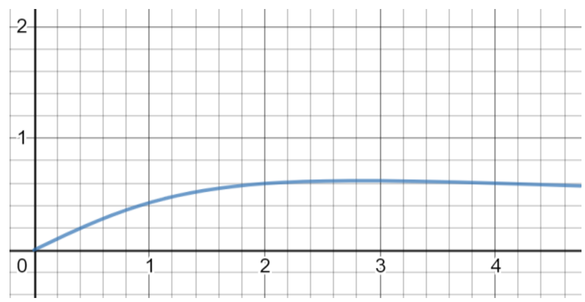

Figure 8. DM contribution to rotation curve.

Bringing the DM and LM contributions together gives us:

$$v=\sqrt{e^{-\frac{1}{2\pi r}}\frac{1/4}{r}+\frac{2r}{r^2+4}\cos(\arctan\frac{2}{r})} \tag{11}$$

which produces this:

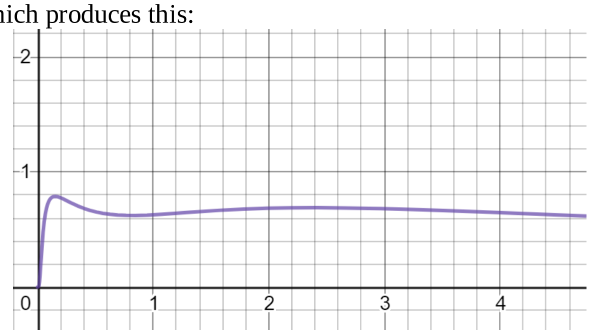

Figure 9. Rotation Curve

As we can see from Figures 7, 8, and nine we see rotation curves similar to what we see in Figure 1 using the DM singularity model.

V. OFF-CENTER CASE

Now we will have the galaxy shift to a new position as it orbits the DM singularity shown in Figure 10:

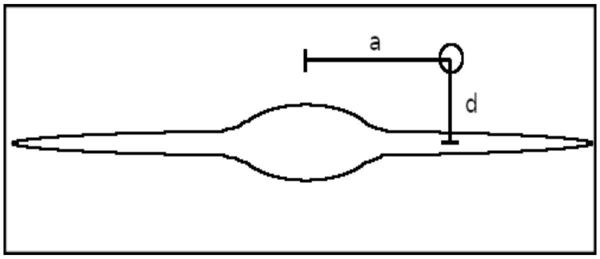

Figure 10. Galaxy as it orbits moving a=2 to the left and d=1 upwards.

The galaxy has now shifted upwards a distance of 1 (d=1) and a distance of 2 to the left (a=2). The DM singularity is now half way between the center of the galaxy to the edge of it, revealing the size of the galactic plane as having a radius of 4.

In this calculation we will resort to an approximation by dividing the galactic plane into four quadrants as shown in Figure 11:

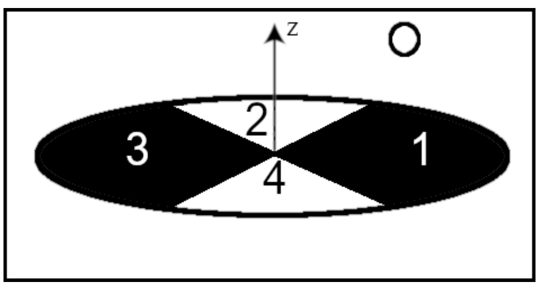

Figure 11. The galactic plane split into four quadrants normal to the x-y plane where the DM singularity sits atop of the first quadrant

The DM singularity is also on top of the axis between the first and third quadrant in addition to being halfway between the center of the plane and the edge. The galactic plane is now normal to the x-y plane in this example. As we delve into these next two cases it will help to visualize the galactic plane as having a field of vectors, although we will not be working with vectors directly.

In the first quadrant, since the symmetrical part of the quadrant is split in the middle due to the position of the DM singularity, the angular components of the force (around the z-axis) point towards each other, making them equal and opposite and canceling each other out eliminating any contribution to the rotation curve. So we will only use the radial vectors of the forces and further approximate it by having the vectors all parallel as if the quadrant is converged into a one-dimensional distribution of mass.

But first we will get the proportion of the quadrants when splitting them in two since the DM singularity is dividing them in the middle: between r=0 and r=2 ($0 \leq r \leq 2$) and r=2 and r=4 ($2 \leq r \leq 4$). Using ¼ πr^2 as the area for the first quadrant we obtain the factor of the area of the first half:

$$\frac{1/4\pi 2^2}{1/4\pi 4^2} = 1/4$$

so $M_{0\leq r\leq 2}$ = ¼ (¼ M_{DM}) and $M_{2\leq r\leq 4}$ = ¼ (¾ M_{DM}), noting that M_{DM} represents the mass contribution the DM has on the rotation curve and not the mass of the quadrant itself. The other factor of ¼ comes from the fact that quadrant 1 is ¼ of the total DM contribution to the curve.

Treating the first quadrant as condensed to a one-dimensional distribution of mass as proposed earlier will give us the contribution of the first half ($0 \leq r \leq 2$), keeping in mind ¼ (¼ M_{DM}) = 1/16 (2M) = 1/8M and now that we have a = 2, then r → 2 - r:

$$a = \sqrt{\frac{G(\frac{1}{8}M)(2-r)}{(2-r)^2 + d^2}\cos\left(\arctan\frac{d}{2-r}\right)} \qquad (12)$$

Before moving on, we must include the centrifugal forces on the contribution of the rotation curve since it is not perpendicular to the disc anymore (the galactic disc now has movement in the y-direction as well as in the x-direction). We will use another approximation and input a simple factor of:

$$\sin\frac{d}{\sqrt{a^2 + d^2}} \qquad (13)$$

In the first example since a = 0, equation (13) just becomes 1 since the centrifugal forces do not influence the contribution of the rotation curve due to the DM singularity. In the next example of the next section we will be looking at the instance the galactic disc is at the apsis (d = 0). So in that case, equation (13) will become 0, meaning that this contribution to the rotation curve will be parallel and opposite as the centrifugal forces completely cancel them out. (Note that using this factor is just a simplified approximation, since the galaxy is a large plane and not a point, and the forces vary with the distance from the DM singularity.) So with that in mind, (12) becomes

$$a = \sqrt{\frac{G(\frac{1}{8}M)(2-r)}{(2-r)^2 + d^2}\cos\left(\arctan\frac{d}{2-r}\right)\sin\frac{d}{\sqrt{a^2 + d^2}}}$$

and using now d = 1, a = 2, G = M = 1, we obtain:

$$a=\sqrt{\frac{(1/8)(2-r)}{(2-r)^2+1}\cos\left(\arctan\frac{1}{2-r}\right)\sin\frac{1}{\sqrt{5}}} \qquad (14)$$

Since this will take away from the orbital velocities of the first half of the quadrant, a negative sign must be used, giving us the plot:

Figure 12. First half of quadrant 1's contribution to the rotation curve.

Which is exactly what we expect on the inner section of the quadrant when the DM singularity is shifted to the right. The second part of the first quadrant, where ¼ (¾ M_{DM}) = 3/16 (2M) = 3/8M and r ⟶ r – 2:

$$b=\sqrt{\frac{G(\frac{3}{8}M)(r-2)}{(r-2)^2+d^2}\cos\left(\arctan\frac{d}{r-2}\right)\sin\frac{d}{\sqrt{a^2+d^2}}}$$

then plugging the known variables we obtain:

$$b=\sqrt{\frac{(3/8)(r-2)}{(r-2)^2+1}\cos\left(\arctan\frac{1}{r-2}\right)\sin\frac{1}{\sqrt{5}}} \qquad (15)$$

Plotting this gives us:

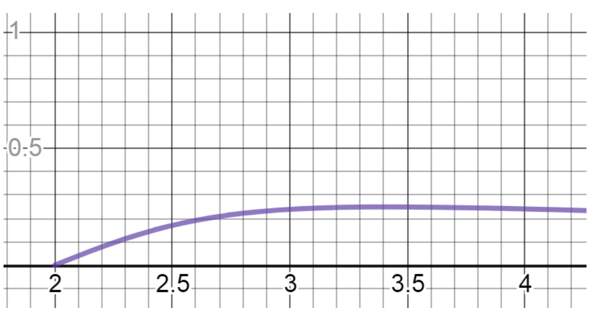

Figure 13. Second half of quadrant 1's contribution to the rotation curve

Next we will calculate quadrants 2 and 4 together. Due to the symmetry, the angular components of the force of one quadrant will cancel the other one since they're equal and opposite. So we will only be including the radial components of the force. And because of the symmetry we will add both quadrants together into our next equation, the contribution being ½ M_{DM} to the rotation curve.

Since the components of the force is perpendicular to the centrifugal forces, a factor of equation (13) is not necessary, and now as the distance becomes $(r^2+d^2+a^2)^{1/2}$ we obtain:

$$c=\sqrt{\frac{G(\frac{1}{2}2M)r}{(r)^2+d^2+a^2}\cos\left(\arctan\frac{\sqrt{d^2+a^2}}{r}\right)}$$

$$c=\sqrt{\frac{r}{(r)^2+5}\cos\left(\arctan\frac{\sqrt{5}}{r}\right)} \qquad (16)$$

keeping in mind that the quadrant is simplified into a one-dimensional distribution of mass, gives us the plot:

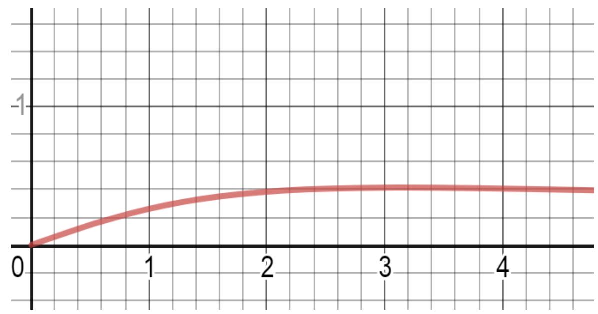

Figure 14. Quadrants 2 and 4's contribution to the rotation curve

The last quadrant to calculate (quadrant 3) with r ⟶ r + 2 and giving it the factor of equation (13) since the centrifugal forces in this case is not perpendicular to the radial components of the force give us:

$$d=\sqrt{\frac{G(\frac{1}{4}2M)(r+2)}{(r+2)^2+d^2}\cos\left(\arctan\frac{d}{r+2}\right)\sin\frac{d}{\sqrt{a^2+d^2}}}$$

$$d=\sqrt{\frac{\frac{1}{2}(r+2)}{(r+2)^2+1}\cos\left(\arctan\frac{d}{r+2}\right)\sin\frac{1}{\sqrt{5}}} \qquad (17)$$

Plotting this gives us:

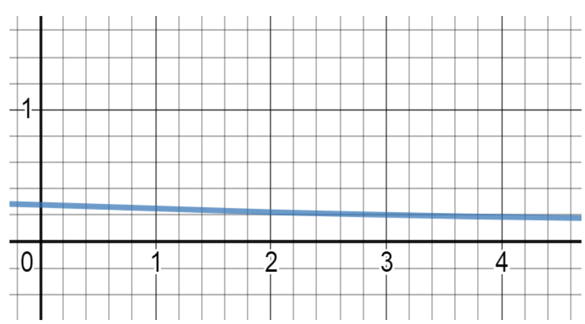

Figure 15. Quadrant 3's contribution to the rotation curve

Putting these equations a,b,c, and d together with the M_{LM} contribution of equation (5) and keeping in mind equation a (eq. 14) contributes negatively to the rotation curve we obtain:

$$v=\sqrt{e^{-\frac{1}{2\pi r}}\frac{1}{4r}-a^2+b^2+c^2+d^2} \quad (18)$$

gives us the plot:

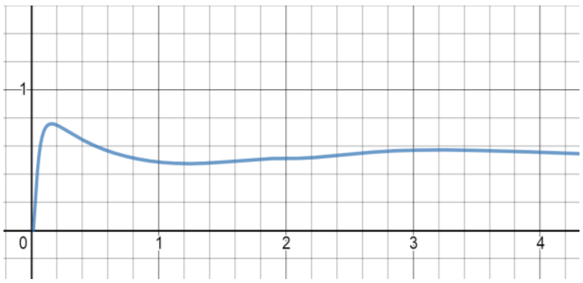

Figure 16. Rotation Curve for off-center case

This plot has a little more variation in the shape of the curve, being more bumpier compared to Figure 9. This case looks more reminiscent to actual rotation curves plotted from actual data.

VI. LAST CASE

The last case will involve the galactic plane reaching the apsis as show in Figure 17:

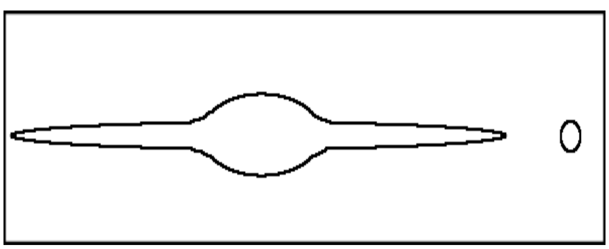
Figure 17. Galactic plane at the apsis of the orbit. DM singularity is now at a = 5 from the center of the galactic plane

In this case, where d = 0, the factor of equation (13) becomes zero. This now eliminates any of the terms from the last case that included the factor and we are left with equation (16) now with a = 5:

$$c=\sqrt{\frac{G(\frac{1}{2}2M)r}{(r)^2+d^2+a^2}\cos(\arctan\frac{\sqrt{d^2+a^2}}{r})}$$

$$c=\sqrt{\frac{r}{(r)^2+25}\cos(\arctan\frac{5}{r})} \quad (19)$$

and this plots as:

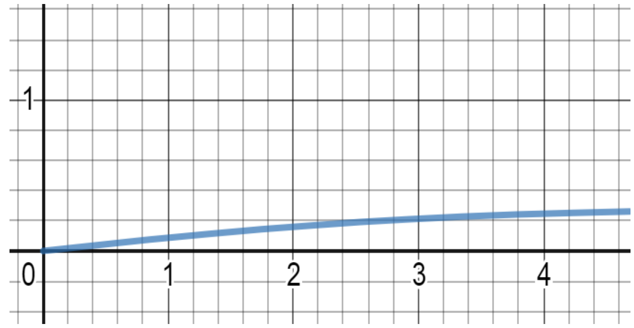

Figure 18. Quadrants 2 and 4's contribution to the rotation curve in the last case

And now (including equation (5) in the plot) we obtain the rotation curve shown in blue:

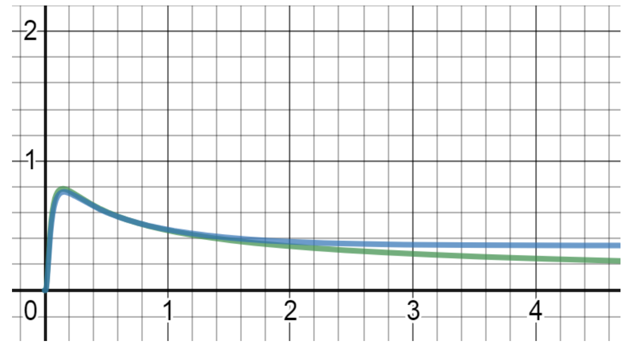

Figure 19. Rotation curve with DM contribution included shown in blue, without DM shown in green.

I included the rotation curve that did not include the DM contribution shown in green to show the comparison between the two. We can she that both curves are similar except in the end where the dark matter starts to make evident its contribution and show its influence and existence even when the galactic plane is at the apsis of the orbit of the DM singularity. Therefore, at any point in the orbit, influence of dark matter is made evident in the rotation curves although the shape varies depending on its position in the orbit. This of course does not include any influence of other DM singularities that are in proximity to the galaxy which if it was can give more complexity to the rotation curves when having more complex systems. But this is beyond the scope of this paper as its just an introduction of the concept of this model. Globular clusters were also disregarded but can be included in future attempts of this model.

VII. CONCLUSION AND FUTURE SCOPE

From the three cases that were presented, we can see how easily these rotation curves manifest themselves under the DM Singularity Model. We can expect that at every instance in our system the DM singularity has influence in the velocities of the stars in the galactic plane. Due to using several approximation methods in the second and third case (sections V and VI) there were certain limitations in modeling those rotation curves in those instances, such as using the factor of equation (13) to include the influence of centrifugal forces and reducing each quadrant to a one-dimensional distribution of mass, where some parts of the quadrant were not reduced to their proper components. But these methods were used in order to be able to approximate these curves on a 2D plot. With better computation and software, better methods can be used to create more accurate models, as well as being able to model more complex systems which could then be used to study real systems and the structures of these galactic clusters.

In the future we can also take into account the varying densities of the galactic plane due to galactic spirals in these models. The study of how globular clusters are influenced using the DM singularity model must also be looked at, and will be the focus for the next work which was not touched upon here.

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