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Urban Water Consumption Estimation Based on Advanced GM (1, 1) Model

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Abstract— In this paper, based on actual water consumption data in a Chinese City, we establish a grey system estimation model about urban water consumption estimation, in order to study the application of the advanced GM (1, 1) model in Chinese urban water supply and demand field. In conclusion, we found it is considered that the advanced GM (1, 1) model can be used to predict the total urban water consumption, besides, based on the comparison with actual water consumption, an accuracy test is also further employed to improve the GM (1, 1) model continuously.

Keywords—Urban, Water Consumption, Estimation, Advanced GM (1, 1)

I. INTRODUCTION

Under the new economic situation for urban development, the overall balance and long-term balance of water supply and demand must be ensured, and forecasting urban water consumption is an important aspect. In this paper, we intend to employ the GM (1, 1) model based on the grey system theory, in order to make a series of predictions on the total urban water consumption in a Chinese city [1].

The reason for adopting this method is that because the total demand for water and its growth are affected by many factors such as economic development, energy structure, industrial structure, residents' income level, and even climate change, some of these factors are certain, and some factors are not [2], [3]. Thus it can be regarded as a grey system; the grey prediction method can avoid the fatal weakness of insufficient data, and also avoid subjective judgments caused by factors such as personal experience, knowledge and preferences, and macro policies, making it can better grasp the self-evolution law of the system [4].

II. GM (1,1) MODEL

GM (1, 1) Model Basic

GM (1, 1) is a first-order, single-variable differential equation model, which is suitable for predicting the development and fluctuation of the behavioural characteristic value for a system. And the essence of the GM model is to generate the accumulation of the original data sequence (1-AG0) in one time, so that the new generated data sequence will be presented within a certain rule, and thus we can construct an estimation model [5], [6], [7], [8].

The raw data used in this study are from the Annual Book of a Chinese city from 2011-2018, as shown in Table 1.

Table 1 Historical Water Consumption Unit: Million t

Year	**2014**	**2015**	**2016**	**2017**	**2018**
Volume	11.8	12	12.3	12.6	13.69

We take 2014 as the starting point, that is, at this point t = 1, so we have the original data sequence:

$$X^{(0)} = \{X^{(0)}(t) | t = 1,2,\cdots,5\}$$
$$= \{X^{(0)}(1), X^{(0)}(2), \cdots, X^{(0)}(5)\}$$
$$= \{11.8, 12, 12.3, 12.6, 13.69\}$$

(1) First, according to the GM (1, 1) model, a first-order accumulation is generated on the known original data sequence $X^{(0)}$ (that is, 1-AGO):

$$X^{(1)}(t) = \sum_{m=1}^{t} X^{(0)}(m),$$

and thus we obtain the sequence $X^{(1)}$ as follows:

$$X^{(1)} = \{X^{(1)}(t) | t = 1,2,\cdots,5\}$$
$$= \{X^{(1)}(1), X^{(1)}(2), \cdots, X^{(1)}(5)\}$$
$$= \{11.8, 23.8, 36.1, 48.7, 62.39\}$$

(2) Then, we construct a matrix B and a vector YN for data

$$B = \begin{bmatrix} -\frac{1}{2}\left(X^{(1)}(1) + X^{(1)}(2)\right) & 1 \\ -\frac{1}{2}\left(X^{(1)}(2) + X^{(1)}(3)\right) & 1 \\ \cdots & 1 \\ -\frac{1}{2}\left(X^{(1)}(n-1) + X^{(1)}(n)\right) & 1 \end{bmatrix} = \begin{bmatrix} -17.8 & 1 \\ -29.95 & 1 \\ -42.4 & 1 \\ -55.545 & 1 \end{bmatrix}$$

$$Y_N = [X^{(0)}(2), X^{(0)}(3), \cdots\cdots, X^{(0)}(n)]^T$$
$$= [12, 12.3, 12.6, 13.69]^T$$

(3) Estimate parameter $\hat{\alpha}$

$$\hat{\alpha} = (a, b)^T = (B^T B)^{-1} B^T Y_N$$
$$= (-0.042980525, 11.0819881)^T$$

(4) Model Establishment:

According to the parameter $\hat{\alpha}$, we construct time response equation of the model:

$$\hat{X}^{(1)}(t+1) = (X^{(0)}(1) - \frac{b}{a})e^{-at} + \frac{b}{a}$$
$$= 269.63743017e^{0.042980525t} - 257.83743017$$

Among them: t = 0, 1, 2,... N.

Advanced GM(1,1) Model

In order to improve the accuracy of the model, secondly, the parameters were estimated by a fit to further improve the model. We re-write the above time response equation as:

$$X^{(1)}(t+1) = A_e^{0.042980525t} + B$$

According to the first estimated value a and the original 1-AGO sequence $X^{(1)}(k)$, we will estimate both A and B. Thus, we construct a matrix G and a vector X (1) for data:

$$G = \begin{bmatrix} e^0 & 1 \\ e^{-a} & 1 \\ \cdots & \cdots \\ e^{-a(n-1)} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1.043917564 & 1 \\ 1.089763881 & 1 \\ 1.137623656 & 1 \\ 1.187585317 & 1 \end{bmatrix}$$

$$X^{(1)} = (X^{(1)}(1), X^{(1)}(2), \cdots, X^{(1)}(5))^T$$
$$= \{11.8, 23.8, 36.1, 48.7, 62.39\}^T$$

We estimate the parameters both A and B:

$$\binom{A}{B} = (G^T G)^{-1} G^T X^{(1)}$$
$$= (268.8934341, -257.0139582)^T$$

Then the final time response equation is obtained:

$$X^{(1)}(t+1) = 268.8934341e^{0.047990025t} - 257.0139582$$

This is a prediction model based on the 1-AGO generation sequence. By the generated inverse accumulation IAGO, we can obtain the prediction model for the original sequence, that is, the prediction model of the total urban water consumption in the city:

$$\hat{X}^{(0)}(t+1) = \hat{X}^{(1)}(t+1) - \hat{X}^{(1)}(t)$$
$$= 268.8934341 \times (e^{0.042980225t} - e^{0.042980525(t-1)})$$

Among them: $t = 0,1,2 \ldots N$, where

$$X^{(0)}(1) = \hat{X}^{(1)}(1)$$

III. ACCURACY TEST

First, we calculate the residuals:

$$e(t) = X^{(0)}(t) - X^{(0)}(t), t = 1,2,\cdots,N$$

And we obtain the residual vector:

$$e\big(e(1), e(2), \cdots, e(N)\big) = \begin{pmatrix} -0.079475908 \\ 0.190855314 \\ -0.027773557 \\ -0.269179345 \\ 0.255637643 \end{pmatrix}$$

Let the variance of the original sequence $X^{(0)}$ and the residual sequence e be S_1^2 and S_2^2, then

$$S_1^2 = \frac{1}{N}\sum_{t=1}^{N} (X^{(0)}(t) - \overline{X}^{(0)})^2 = 0.440736$$

$$S_2^2 = \frac{1}{N}\sum_{t=1}^{N} (e(t) - \overline{e})^2 = 0.036067974$$

We calculate the posterior error ratio:

$$C = \frac{S_2}{S_1} = 0.286 \leqslant 0.35,$$

Therefore, accuracy level for the model is level l (good);

We also calculate the error probability:

$$P = P\{|e(t) - e| < 0.6745S_1\} = \frac{5}{5} = 100\% \geqslant 95,$$

therefore, the model accuracy level is level 1 (good).

In summary, the accuracy of this model is level 1 (good).

The posterior error test of this model shows that the model is reliable, has high prediction accuracy. Besides, it is not necessary to establish a GM (1, 1) residual correction model, and it still has high prediction accuracy.

Using the model established above, the historical data in Table 1 are tested, and the estimation results and error analysis are shown in Table 2.

Table 2 Accuracy Test

Year	2014	2015	2016	2017	2018
Actual	11.8	12	12.3	12.6	13.69
Estimation	11.88	11.81	12.33	12.87	13.43
Absolute Error	0.080	-0.191	0.028	0.269	-0.256
Relative Error	0.67%	-1.59%	0.23%	2.14%	-1.87%

IV. URBAN WATER CONSUMPTION ESTIMATION

The urban water consumption forecast model established in this paper, to forecast urban water consumption in the future from 2020 to 2026 in a Chinese city.

The prediction is based on 2013 as the origin, that is, at this point t = 1, and the parameter t in each subsequent year is accumulated in turn, and the results are: 14.02 million tons in 2019, 14.63 million tons in 2020, 15.28 million tons in 2021, 15.95 million tons in 2022, 16.66 million tons in 2023, 17.39 million tons in 2024, and 18.15 million tons in 2025, and 18. 95 million tons in 2026.

V. CONCLUSION

Modelling with grey system theory can overcome the deficiency of related data and avoid the influence of human factors. The simulation results of the prediction model established in this paper are close to the actual values, the prediction errors are small, and the model accuracy is also good, which shows that the prediction method based on the grey theory is suitable for predicting the trend of total urban water consumption. This method is practical and effective, and can be employed as an important reference for the supply planning of water industry.

However, for the reason that the parameter a in the model is regarded as a constant when estimation, the grey forecast is not suitable for medium and long-term forecasting. The further in the future, the greater the error of the estimation value. For further study, the parameters *a* and *b* should be regarded as a function of time *t*. As time goes by, the parameters a and b should be continuously predicted and modified, and the original sequence is predicted by the revised numerical substitute GM (1, 1) method.

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