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Research Article

Heat Source/Sink Along Mass Suction Impacts on the Flow of the MHD Boundary Layer across a Flat Plate

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Abstract— in this study, we examined how a boundary layer fluid flow toward a flat plate is affected by the combined actions of a heat source/sink and mass suction. Partial differential equations (PDEs) are used to illustrate the flow sensation. Making use of the proper similarity transformation technique, the system of ODEs is derived from the PDEs. The shooting method is then applied to these updated equations. According to the analysis, a Lorentz force is produced by magnetohydrodynamic (MHD) fluxes, and this effect is amplified by a greater Suction parameter, which compresses the temperature profile. Moreover, as the heat source/sink variable is increased, the temperature profile gets better. Additionally, the study demonstrates that the ambient temperature of the dense dissipative fluid increases as the Eckert number grows. The results may find use in a variety of technical domains, including the optimization of petroleum pipeline flow. The findings can direct further research in this field and advance our understanding of heat and mass transfer phenomena.

Keywords— Heat source/ sink, boundary layer, Suction, MHD, heat and mass transfer, Magnetic field, Chemical reaction.

1. Introduction

Boundary layer flow is a crucial feature in many fluid dynamics scenarios, from environmental systems to industrial operations. Optimizing heat and mass transfer processes and improving fluid flow efficiency in a variety of engineering applications require a thorough understanding and ability to manipulate boundary layer behavior. Researchers and engineers alike have become very engrossed in studying the impact of external elements, like magnetic fields and chemical reactions on boundary layer flow in recent years. When a chemical concentration and temperature are known, chemical reactions happen at a predictable rate. The process of mixing multiple distinct chemical processes to produce the item of interest is known as chemical synthesis. In biochemistry, a variety of chemical processes create metabolic pathways. A first-order reaction's rate depends on the concentration of a single component. This inquiry considers the first-order response. Heat and mass transfer analysis is an exciting field of study due to its potential applications in nuclear power, combustion modeling, heat exchangers, and cooling system design, as well as a variety of aviation propulsion technologies, chemical engineering, and electronics. Thermal radiation heat transmission greatly influences the properties of heat transfer in high-temperature situations. The speeds of chemical reactions can be calculated using heating and the number of substances present. Several chemical reactions are combined in chemical synthesis to produce the end product. Chemical reactions are connected to form metabolic pathways in biochemistry. The first-order solution is the main subject of this analysis. The following processes rely on combined heat and mass transfer issues in the presence of chemical reactions: drying, heat and moisture distribution over agricultural fields and orchards, crop damage from the freezing point, evaporated at the surface of a water body, energy transfer in a wet cooling processes tower, and movement in a desert cooler. Many investigators have studied mass and heat transfer in fluids undergoing chemical reactions [1–15].

In order to take into account, the effects of a magnetic field and a chemical reaction, Desale and Pradhan [16] as well as Oyelami and Falodun [17] explored the heat and mass transfer of magnetohydrodynamic boundary layer flow along a flat plate. Kang et al. [18] as a consequence devised numerous methods for improving the thermal resistance of these materials by incorporating Nano-sized particulate particles into liquid. Because nanometer-sized materials offer exceptional mechanical properties. Nanotechnology is frequently utilized in commercial processes, Choi et al. [19] and [20] revealed that a little quantity of nanoparticles (less than 1% by volume) increased the thermal conductivity of typical heat exchange liquids by about two times. Khanafer et al. [21] appears to be one of the researchers to look at the heat transmission characteristics of nanofluids within the enclosure while taking nanoparticle dispersion into account. Based on these evidences, many scientists believed that nanotechnology will be one of the fundamental variables powering the next major industrial revolution in this century. The researchers employed the most basic border conditions possible, such as maintaining a constant temperature and nanoparticle proportion along the boundary. The Minkowycz et al. [22], discussed on the problem of spontaneous convection across a vertical plate in a porous material saturated with a nanofluid, which was further investigated by Nield and Kuznetsov [23]. The nanofluid idea accounts for Brownian motion and thermophoresis properties and the porous structure was simulated using the Darcy model.

Ghasemi et al. [24] used the probative quadrature method to examine the impact of a nanofluid auxiliary than a stretchable sphere on the magnetization. Ibrahim et al. [25] looked at how a magnetic field affected flow at the stagnation point and thermal expansion from a nanofluid to an expanding sheet. To solve the governing equations numerically, the researchers employed the fourth-order Runge-Kutta method with the firing approach. Wakif et al. [26] offered a novel simple mechanism for resolving the issue. Li et al. [27] studied the thermally stability of a biphasic heterogeneous hybridized nanofluid with corresponding volumetric percentages of Al_2O_3 and CuO nanoparticles in the aqueous solution in a confined area. Thumma et al. [28] looked at the convective MHD nanofluids boundary-layer continuous movement of beyond just a non- linearly pitched stretching / shrinking sheet while taking viscous dissipation into account. The method of differential Transformation was applied by Ghasemi et al. [29] to investigate the DTM (heat generation and thermal conductivity).

Based on these propositions, Helen et al. [30] expanded the mass and heat transfer problem of Desale and Pradhan [31] in addition to Oyelami and Falodun [27] to take into consideration the effects of chemical reaction and a magnetic field. They focused on the local skin friction coefficient, local Nusselt number, and local Sherwood number while analyzing the effects of all flow parameters on velocity, temperature, and concentration profiles. The expected outcomes of this study should provide insightful information about how chemical interactions and magnetohydrodynamic (MHD) fluxes influence the creation of a Lorentz force, which in turn affects the boundary layer flow's velocity profiles. Furthermore, their analysis demonstrated how the magnetic parameter affected the Lorentz force's magnitude, which finally resulted in a compacted velocity profile. The effect of the chemical interaction variable in enhancing the velocity profile provided more evidence for their conclusions. The study also tried to look at how the thick dissipative fluid's ambient temperature was affected by the Eckert number, a crucial variable that characterizes the viscous dissipation effect. Among the many technological applications that these insights have the potential to revolutionize is the improvement of petroleum pipeline flow.

Madaki et al. [32, 33, 34] have conducted a thorough discussion on the idea of magnetohydrodynamics compression flow of vanadium pentoxide (V2O5)- They took into account the impacts of heat radiation, chemical reactions, heat generation and absorption, Hartman numbers, Deborah numbers, and a number of other characteristics based on Jeffrey Hybrid Nanofluid.

Hussaini [35] discussed on the influence of heat generation/ absorption along suction/ injection over a Powell- Eyring.
The main purpose of this research is to investigate the effects of heat transfer on MHD fluid flow over a shrinking sheet by addressing the research gap in the combined effects of heat source/sink and mass suction. The numerical solution of the coupled non-linear momentum, energy and nanoparticle concentration equations is obtained using the fourth-order Runge-Kutta method with the shooting technique. The velocity, temperature and nanoparticle concentration are computed and the results obtained are depicted graphically, for different pertinent parameters. The goal of this research is to advance scientific understanding, particularly in the fascinating field of fluid dynamics and related fields, and to provide new avenues for creative engineering solutions.

2. Experimental Procedure

A continual flow of a fluid that resembles a velocipede across a flat plate is taken into consideration, and the temperature and concentration of the entire surface are kept constant. The fluid has a concentration of C_∞ and a temperature of T_∞ at the open channels. It is believed that a homogeneous first-order reaction with a constant rate k_c is responsible for the chemical interaction between the liquid and the diffusing components.
It is thought that the fluid's characteristics never change. We can ignore the induced magnetic field by taking a weak magnetic Reynolds number. The magnetic field is continually applied at maximal intensity B_o. Viscous dissipation is thought to have a major effect. The surface temperature of the flat plate is thought to fluctuate, as seen in picture 1 below. Assuming the boundary layer approximation is valid, the following are the governing equations for the Boussinesq approximation.

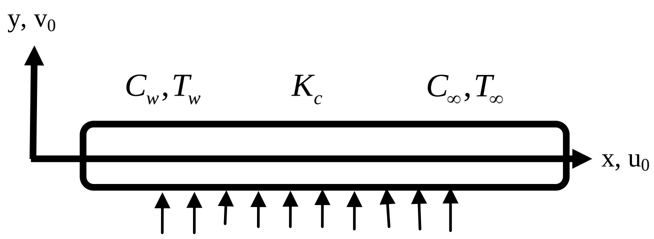

Figure1 The physical model of the problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u, \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho Cp}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho_{nf}}\left(T-T_\infty\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_c\left(C-C_\infty\right), \tag{4}$$

The constraints are stated as follows:

$$\begin{aligned}&u=0, T=T_w(x), C=C_w(x)\,at\,y=0,\\&u=U, T=T_\infty, C=C_\infty\,as\,y\to\infty\end{aligned} \tag{5}$$

In this case, ϑ denotes the coefficient of viscosity, α denotes the thermal conductivity of the fluid, Q is the radiative heat transfer, T is the fluid's temperature, C denotes the fluid's concentration, and σ implies the fluid conductivity. In addition, u and v represent the fluid's velocities in the x and y directions, while B_0 represents the electromagnetic induction. The specific heat at constant pressure is denoted by C_p, the fluid density is implied by ρ, the mass diffusivity is represented by D, the coefficient of chemical interactions is shown by k_c, and the heat at the free stream is indicated by T_∞. The concentration at ambient is indicated by C_∞, the wall's heat is indicated by T_w, and the wall's concentration is indicated by C_w. Assuming that the plate's temperature varies in the manner described below.

$$T_w(x)-T_\infty=Ax^n \tag{6}$$

Temperature $\theta(\eta)$ and concentration $\varphi(\eta)$ are used as a stream term ψ and a similarity term η

$$\begin{aligned}&\eta=y\sqrt{\frac{U}{vx}}, \varphi(\eta)=\frac{C-C_\infty}{C_w(x)-C_\infty}, \psi=\sqrt{Uvx}\,f(\eta),\\&\theta(\eta)=\frac{T-T_\infty}{T_w(x)-T_\infty}\end{aligned} \tag{7}$$

$$U=\frac{\partial\psi}{\partial y}=Uf'\,and\,v=-\frac{\partial\psi}{\partial x}=\frac{1}{2}\sqrt{\frac{Uv}{x}}\left(\eta f'-f\right) \tag{8}$$

It is noteworthy that prime denotes differentiation about η. The following is the result of reducing Eq. (1) to Eq. (5) using Eqs. (6) to (8):

$$f'''+\frac{1}{2}ff''-M^2 f'=0, \tag{9}$$

$$\theta''+\frac{1}{2}f\theta'+Ec\Pr\left(f''\right)^2-n\Pr f'\theta+\lambda_1\theta=0, \tag{10}$$

$$\varphi''+\frac{1}{2}f\varphi'+Sc\lambda_2 f'\varphi-Sc\lambda\varphi=0, \tag{11}$$

The following boundary conditions apply:

$$\begin{aligned}&f'(0)=0, f(0)=S, \theta(0)=1, \varphi(0)=1\\&f'(\infty)=1, \theta'(\infty)=0, \varphi'(\infty)=0,\end{aligned} \tag{12}$$

Where $M=\frac{v\sigma B_0^2}{\rho v_0^2}$ represents the magnetic phrase, $\Pr=\frac{v}{\alpha}$ indicates the overall amount of Prandtl number, $Ec=\frac{U^2}{C\left(T_w(x)-T_\infty\right)}$ stands for the total amount of Eckert number, whereas n is the surface temperature parameter, Sc = ν/D represents the Schmidt number, $\lambda_1=\frac{Q}{UT_w\rho_{nf}}$ is the heat source/ sink parameter, $\lambda_2=\frac{l^2\gamma}{v}$ is the chemical reaction parameter, S is the suction parameter.

3. Results and Discussion

Equations (9- 11) was numerically solved with Maple 14.0, taking into account the boundary conditions. This software solves boundary value problems numerically by default using the fourth-fifth order Runge-Kutta- Fehlberg approach. We verified its correctness and robustness by contrasting our findings with published findings. These findings apply to a scenario in which the fluid is a regular fluid devoid of nanoparticles and the flat plate is kept at a constant temperature in the absence of any heat source, sink, or mass suction effects.

Temperature Profile

Figure 2 depicts the impact of varying Eckert number parameter (Ec) on the temperature profile. An increment in Eckert's number implies an increase in the kinetic energy of the fluid particles, which increases the vibration of particles and leads to collisions. Due to this collision, the dissipation of heat in the boundary layer's region increases the fluid temperature. A closer look at Figure 3 illustrates how the temperature distribution is rising together with the enhancement of the heat source/sink parameter (λ1) values. Physically, the average kinetic energy of the fluid particles causes an increase in the temperature distribution field transit from heat absorption (λ1 < 0) to heat generation (λ1 > 0), this is because the temperature affects the kinetic energy involved in the movement of the fluid molecules and nanoparticles, which also results in tiny gaps between the fluid molecules, which causes an increase in the movement and speed of the fluid molecules within the channel. Figure 4 shows the influence of the surface temperature parameter (n) over the temperature profile. On the other hand, when n grew, there was a slight decrease in the fluid's temperature.

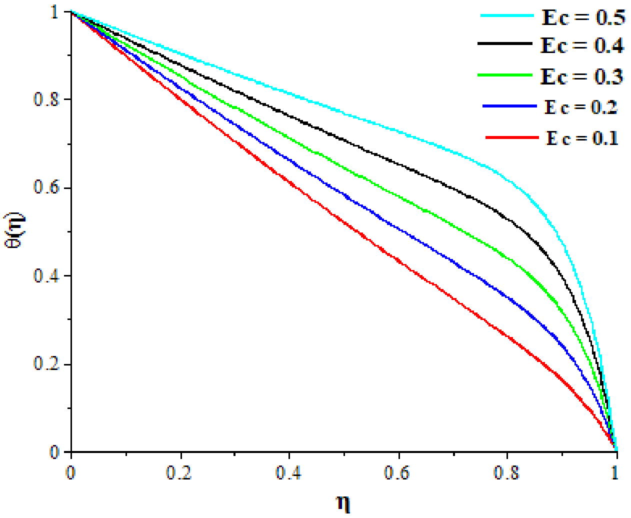

Fig.2: Effects of Eckert number (Ec) on temperature profile

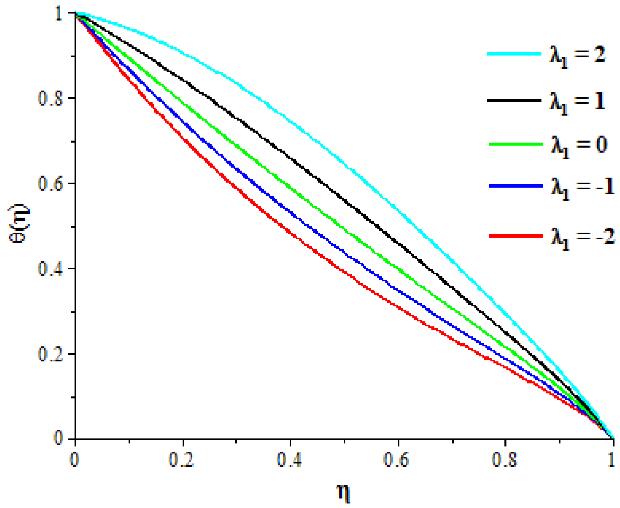

Fig.3: Effects of Heat source/ sink parameter (λ_1) on temperature profile

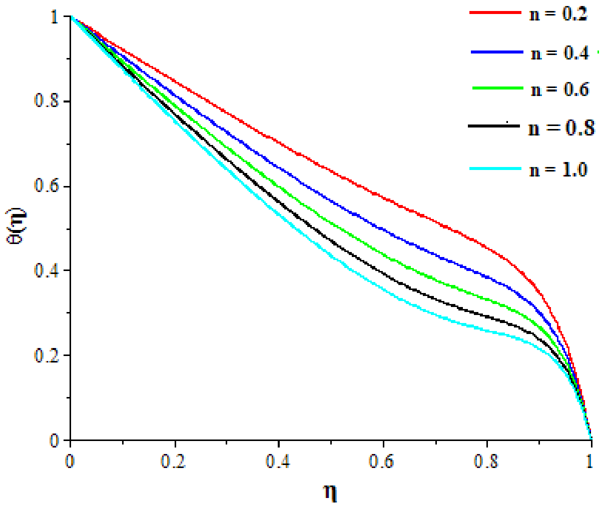

Fig.4: Effects of surface temperature parameter (n) on temperature profile

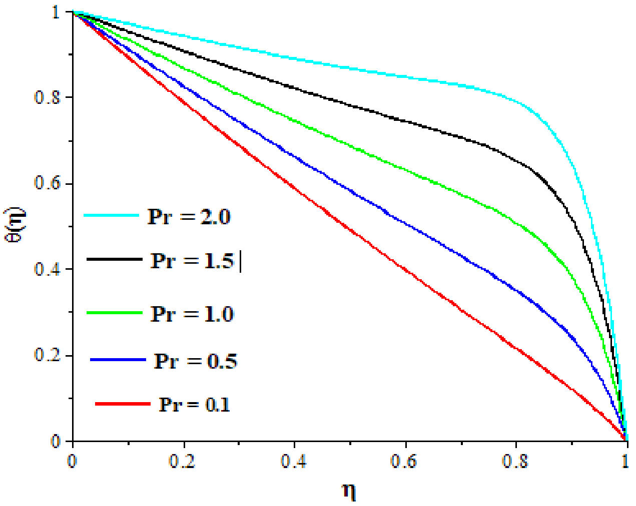

Fig.5: Effects of Prandtl number (Pr) on temperature profile

Figure 5 depicts the effects of the Prandtl number parameter (Pr) over the temperature profile, it can be observed that any increase in Pr, raises the temperature of the fluid. Demonstration of the effects of the suction parameter (S) on the temperature profile is categorically achieved in Figure 6. The presence of a porous medium causes a higher restriction to the fluid flow, which in turn slows its motion. Therefore, with increasing suction parameter, the resistance of the fluid to heat increases and hence temperature decreases.

Concentration Profile

The influence of the Suction parameter (S) on the profile of concentration is illustrated in Figure 7. One conclusion we can get from these results is that the wall mass suction is very significant in maintaining the steady boundary layer near the plate by delaying the separation. In this figure, we can also observe that as the values of S increase the nanoparticle concentration of fluid is decreased drastically.

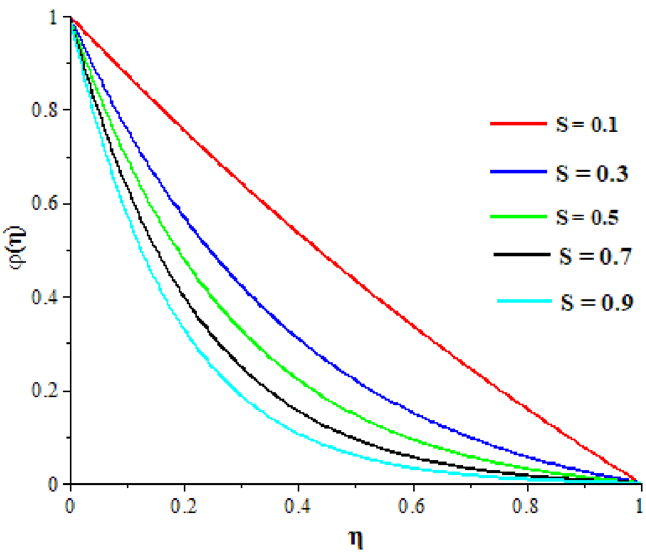

Fig.7: Effects of Suction parameter (S) on concentration profile

However, the Schmidt number is used to characterize fluid flow in which there are simultaneous momentum and mass diffusion processes. It physically relates the relative thickness of the Hydrodynamic layer and mass transfer boundary layer. The effect of the Schmidt number on concentration and concentration gradient has also been studied through Figure 8, in this case, it has been observed that the nanoparticle concentration increases with an increase in the Schmidt number and shear stress profiles decrease drastically with an increase in the Schmidt number and then increases asymptotically. Figure 9 expatiated categorically clear on the influence of chemical reaction parameter (λ2) over the concentration profile. It can be observed that as the parameter increases the nanoparticle concentration profile decreases.

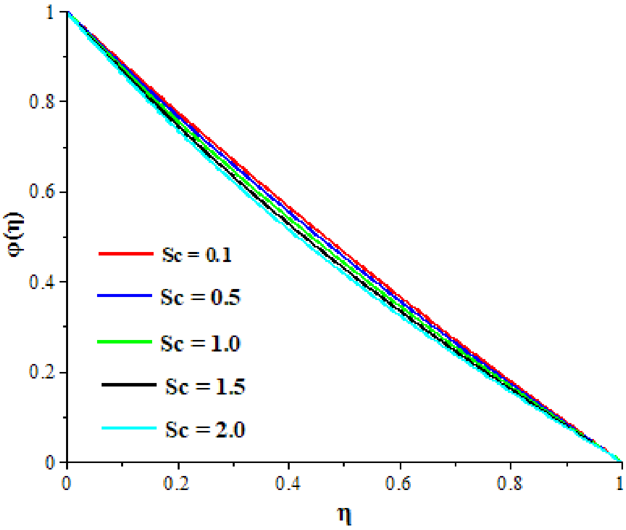

Fig.8: Effects of Schmidt number (Sc) on concentration profile

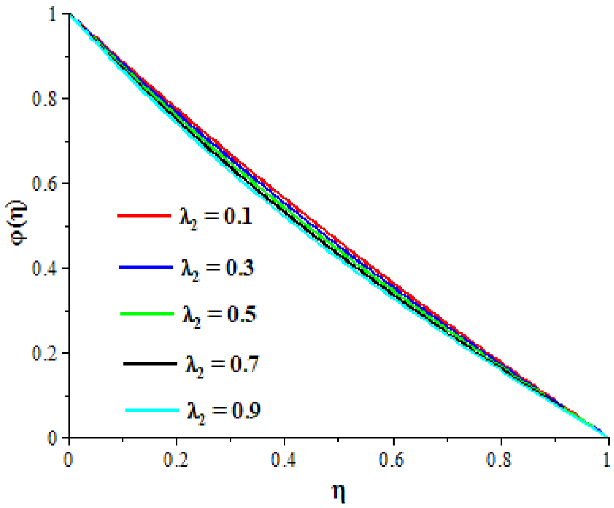

Fig.9: Effects of chemical reaction parameter (λ_2) on concentration profile

Nusselt Number Profile

The Nusselt number profile for various values of heat source/ sink parameter λ1 is shown in Figure 10. We observed that the Nusselt number increases with increasing values of λ1. This is due to the decrease in the thickness of the thermal boundary layer for an increase of the heat sink parameter (λ1 < 0), but it increases with the heat source parameter (λ1 > 0). The effects of the Suction parameter (S) over the Nusselt number profile are depicted in Figure 11, it can be observed obviously that as the parameter increases it brings about a decrease in the Nusselt number profile.

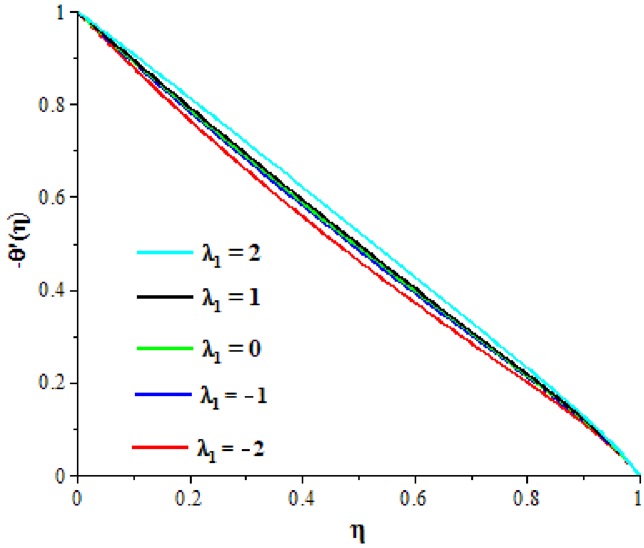

fig.10: Effects of heat source/ sink (λ_1) on Nusselt number profile

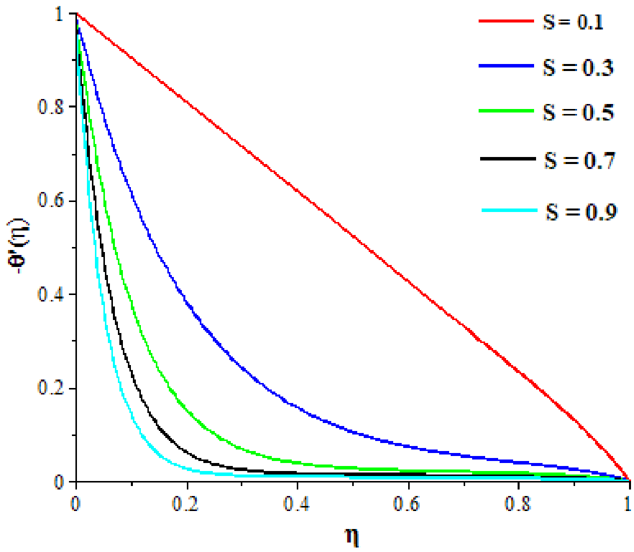

Fig.11: Effects of Suction parameter (S) on Nusselt number profile

Sherwood Number Profile

Figure 12 is plotted to explain the effect of the mass Suction parameter (S) over the local Sherwood number profile. Here we have studied that any increment in the values of the parameter brings about an increase in skin friction coefficient and a reverse for the case of the local Sherwood number. Physically speaking, this seems to be reasonable. It's noteworthy to notice that for both parameters—skin friction and the local Sherwood number for the Suction parameter—the fluid in the presence of nanoparticles is larger than the fluid in its absence. Furthermore, Figure 13 represents

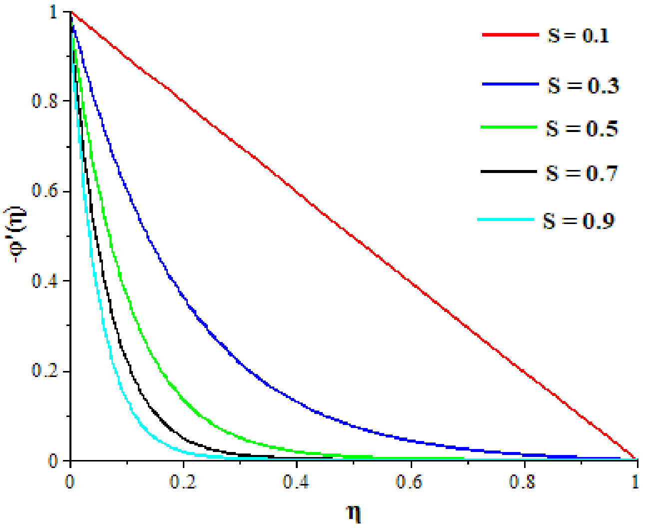

Fig.12: Effects of Suction parameter (S) on Sherwood number profile

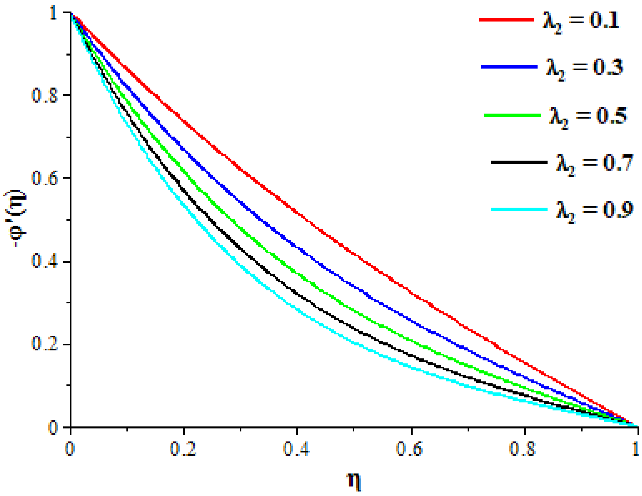

Fig.14: Effects of chemical reaction parameter (λ_2) on Sherwood number profile

The impacts of Schmidt number (Sc) on Sherwood number profile. The profiles of Sherwood number behave erratically as Sc increases. The Sherwood number exhibits extemporaneous comportment. The occurrence of heated air near the powdered may perhaps be to culpability. When the external temperature is nullified, Sc intensifies the Sherwood number profile. Figure 14 displays the fluctuation of the Sherwood number profile with chemical reaction parameter λ2. While the fluid's temperature and velocity do not significantly change when chemical reaction parameters grow, it is noticed that an increase in chemical reaction parameter value reduces the Sherwood number of species in the boundary layer. This is due to the fact that the chemical reaction in this system causes the chemical to be consumed, which lowers Sherwood's number profile. The primary outcome is that the first-order chemical reaction tends to reduce overshoot in the solute concentration profile in the solutal border layer.

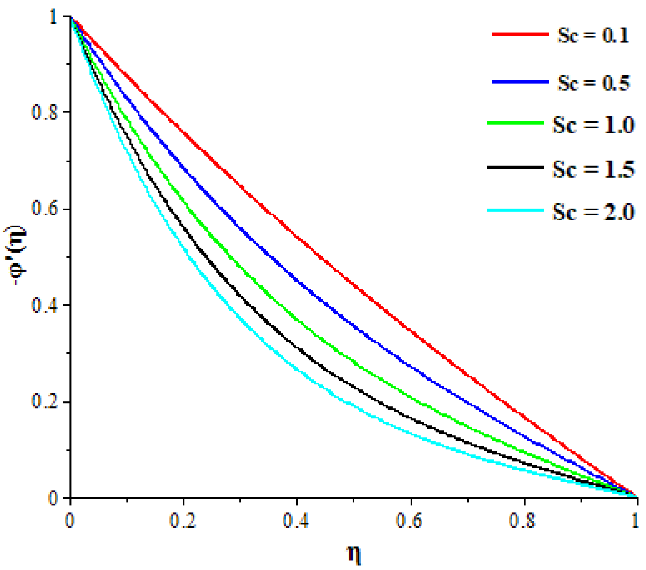

Fig.13: Effects of Schmidt number (Sc) on Sherwood number profile

4. Conclusion and Future Scope

An analytical investigation of the flow boundary layer in a nanofluid caused by the movement of a level plate has been executed. This work is more comprehensive and newer since it uses a convective heating boundary condition rather than a constant temperature or heat flow. The shooting technique was applied to solve the flow equations numerically. The acquired result leads to the following conclusions.

I. Temperature increases with a rise in Ec, λ_1 and Pr. The reverse is the case with n and S.
II. The nanoparticle concentration profile is raised whenever there is an increase in Sc, whereas it declines with S and λ_2.
III. Nusselt number is increased with λ_1, while it decreases with S.
IV. Sherwood number is totally a decreasing function with S, Sc and λ_2.

In conclusion, our study has not only filled a significant knowledge vacuum regarding boundary layer flow involving heat sinks and sources, chemical reactions, and suction effects, but it also has promise for future use in a range of scientific and technical fields. By taking into account the impacts of thermal radiation or diffusion effects, the results direct future study and add to the body of knowledge in the field of heat and mass transfer. It also provides workable answers for streamlining industrial procedures and fluid flow systems.

Data Availability
None.

Conflict of Interest
Author (s) declare that they do not have any conflict of interest.

Funding Source
None

Authors' Contributions
All author(s) contributed equally in the manuscript.

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